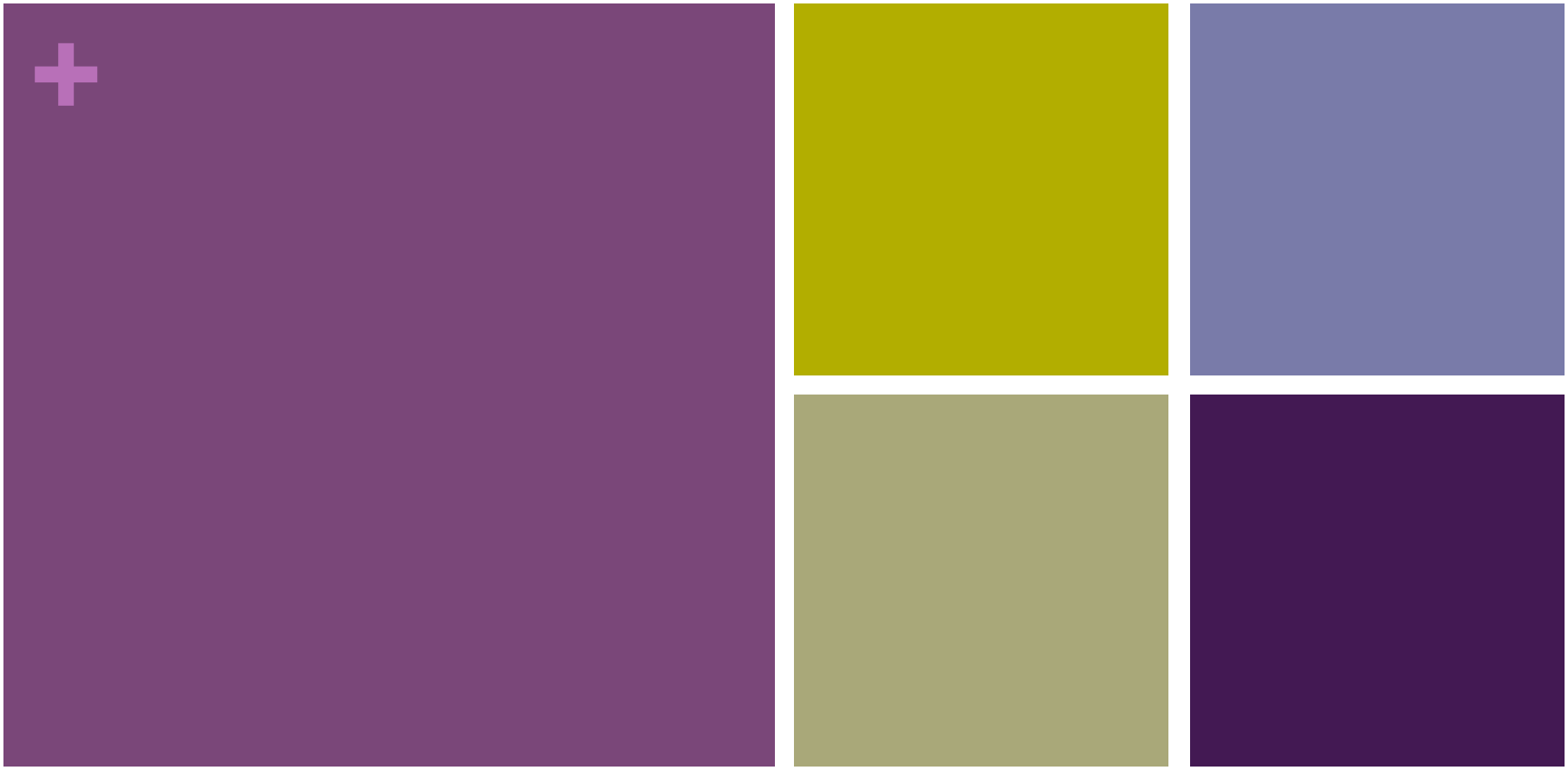


Design of Parallel Algorithms

Models of Parallel Computation

Preliminaries: Decomposition, Tasks, and Dependency Graphs

- The first step in developing a parallel algorithm is to decompose the problem into tasks that can be executed concurrently
- A given problem may be docomposed into tasks in many different ways.
- Tasks may be of same, different, or even interminate sizes.
- A decomposition can be illustrated in the form of a directed graph with nodes corresponding to tasks and edges indicating that the result of one task is required for processing the next. Such a graph is called a *task dependency graph*.

Degree of Concurrency

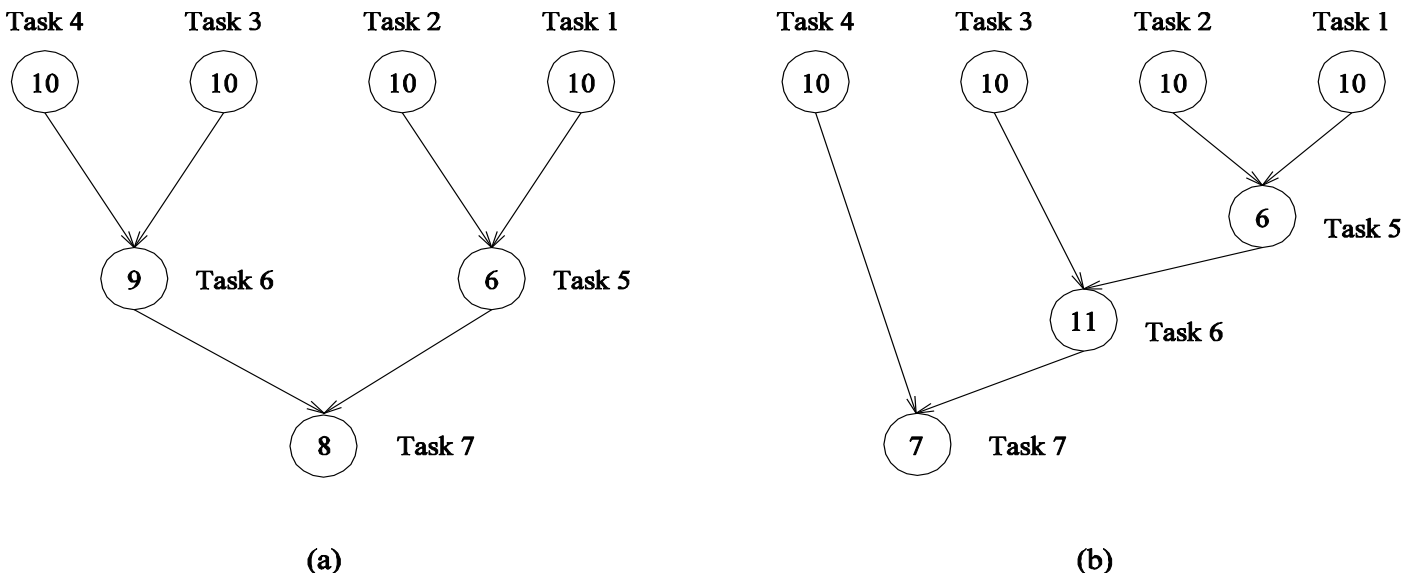
- The number of tasks that can be executed in parallel is the *degree of concurrency* of a decomposition.
- Since the number of tasks that can be executed in parallel may change over program execution, the *maximum degree of concurrency* is the maximum number of such tasks at any point during execution. *What is the maximum degree of concurrency of summing n numbers?*
- The *average degree of concurrency* is the average number of tasks that can be processed in parallel over the execution of the program. *Assuming that each tasks in the database example takes identical processing time, what is the average degree of concurrency in each decomposition?*
- The degree of concurrency increases as the decomposition becomes finer in granularity and vice versa.

Critical Path Length

- The Task Dependency Graph is a directed graph that describes the flow of information between parallel tasks in the program. Because of this dependency, some tasks may not run concurrently with other tasks.
  - A directed path in the task dependency graph represents a sequence of tasks that must be processed one after the other.
  - The longest such path determines the shortest time in which the program can be executed in parallel.
  - The length of the longest path in a task dependency graph is called the critical path length.

Critical Path Length

Consider the task dependency graphs of the two database query decompositions:



What are the critical path lengths for the two task dependency graphs? If each task takes 10 time units, what is the shortest parallel execution time for each decomposition? How many processors are needed in each case to achieve this minimum parallel execution time? What is the maximum degree of concurrency?

Limits on Parallel Performance

- It would appear that the parallel time can be made arbitrarily small by making the decomposition finer in granularity.
- There is an inherent bound on how fine the granularity of a computation can be. *For example, in the case of multiplying a dense matrix with a vector, there can be no more than* (n^2) *concurrent tasks.*
- Concurrent tasks may also have to exchange data with other tasks. This results in communication overhead. The tradeoff between the granularity of a decomposition and associated overheads often determines performance bounds.

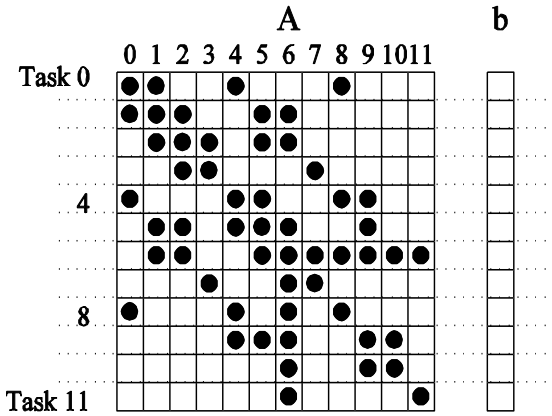
Task Interaction Graphs

- Task interaction graphs are undirected graphs that show data communication patterns between tasks.
  - Represents data communication within the parallel program
  - Subtasks generally exchange data with others in a decomposition. For example, even in the trivial decomposition of the dense matrix-vector product, if the vector is not replicated across all tasks, they will have to communicate elements of the vector.
  - The graph of tasks (nodes) and their interactions/data exchange (edges) is referred to as a *task interaction graph*.
- Note that *task interaction graphs* represent data dependencies, whereas *task dependency graphs* represent control dependencies.

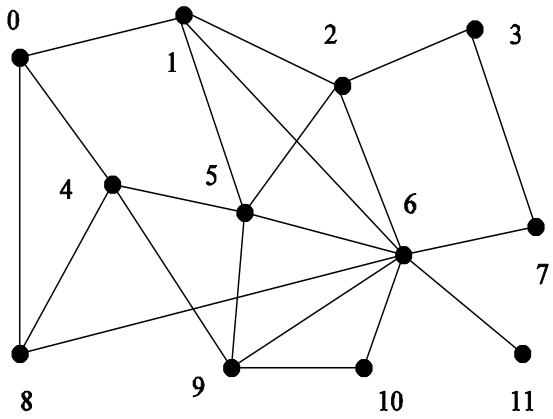
Task Interaction Graphs: An Example

Consider the problem of multiplying a sparse matrix ***A*** with a vector ***b***. The following observations can be made:

- As before, the computation of each element of the result vector can be viewed as an independent task.
- Unlike a dense matrix-vector product though, only non-zero elements of matrix ***A*** participate in the computation.
- If, for memory optimality, we also partition ***b*** across tasks, then one can see that the task interaction graph of the computation is identical to the graph of the matrix ***A*** (the graph for which ***A*** represents the adjacency structure).



(a)



(b)

Task Interaction Graphs, Granularity, and Communication

In general, if the granularity of a decomposition is finer, the associated overhead (as a ratio of useful work associated with a task) increases.

Example: Consider the sparse matrix-vector product example from previous foil. Assume that each node takes unit time to process and each interaction (edge) causes an overhead of a unit time.

Viewing node 0 as an independent task involves a useful computation of one time unit and overhead (communication) of three time units.

Now, if we consider nodes 0, 4, and 5 as one task, then the task has useful computation totaling to three time units and communication corresponding to four time units (four edges). Clearly, this is a more favorable ratio than the former case.

Processes and Mapping

- In general, the number of tasks in a decomposition exceeds the number of processing elements available.
- For this reason, a parallel algorithm must also provide a mapping of tasks to processes.

Note: We refer to the mapping as being from tasks to processes, as opposed to processors. This is because typical programming APIs, as we shall see, do not allow easy binding of tasks to physical processors. Rather, we aggregate tasks into processes and rely on the system to map these processes to physical processors. We use processes, not in the UNIX sense of a process, rather, simply as a collection of tasks and associated data.

Processes and Mapping

- Appropriate mapping of tasks to processes is critical to the parallel performance of an algorithm.
- Mappings are determined by both the task dependency and task interaction graphs.
- Task dependency graphs can be used to ensure that work is equally spread across all processes at any point (minimum idling and optimal load balance).
- Task interaction graphs can be used to make sure that processes need minimum interaction with other processes (minimum communication).

Processes and Mapping

An appropriate mapping must minimize parallel execution time by:

- Mapping independent tasks to different processes.
- Assigning tasks on critical path to processes as soon as they become available.
- Minimizing interaction between processes by mapping tasks with dense interactions to the same process.

Note: These criteria often conflict with each other. For example, a decomposition into one task (or no decomposition at all) minimizes interaction but does not result in a speedup at all! Can you think of other such conflicting cases?

Decomposition Techniques

So how does one decompose a task into various subtasks?

While there is no single recipe that works for all problems, we present a set of commonly used techniques that apply to broad classes of problems. These include:

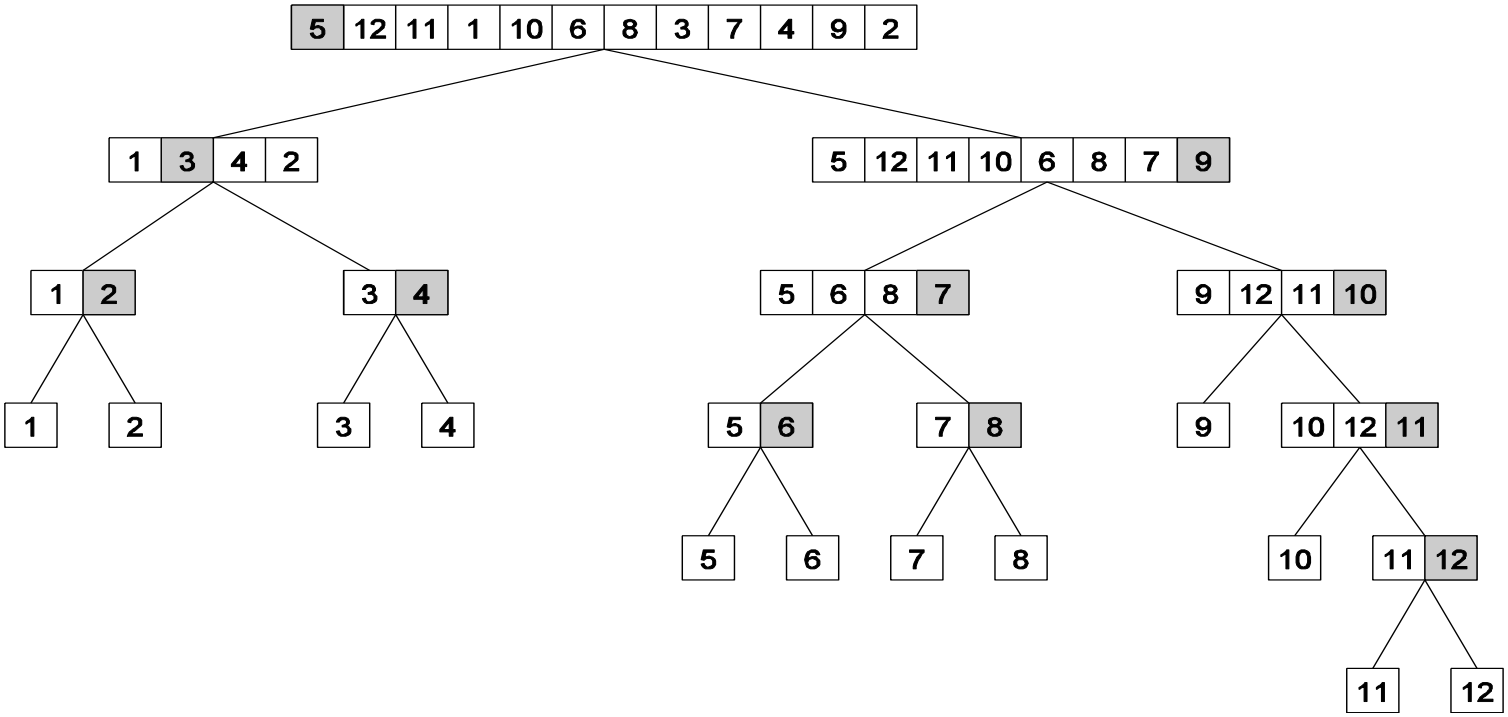
- recursive decomposition
- data decomposition
- exploratory decomposition
- speculative decomposition

Recursive Decomposition

- Generally suited to problems that are solved using the divide-and-conquer strategy.
- A given problem is first decomposed into a set of sub-problems.
- These sub-problems are recursively decomposed further until a desired granularity is reached.

Recursive Decomposition: Example

A classic example of a divide-and-conquer algorithm on which we can apply recursive decomposition is Quicksort.



In this example, once the list has been partitioned around the pivot, each sublist can be processed concurrently (i.e., each sublist represents an independent subtask). This can be repeated recursively.

Recursive Decomposition: Example

The problem of finding the minimum number in a given list (or indeed any other associative operation such as sum, AND, etc.) can be fashioned as a divide-and-conquer algorithm. The following algorithm illustrates this.

We first start with a simple serial loop for computing the minimum entry in a given list:

```
1. procedure SERIAL_MIN (A, n)
2. begin
3. min = A[0];
4. for i := 1 to n − 1 do
5.                 if (A[i] < min) min := A[i];
6. endfor;
7. return min;
8. end SERIAL_MIN
```

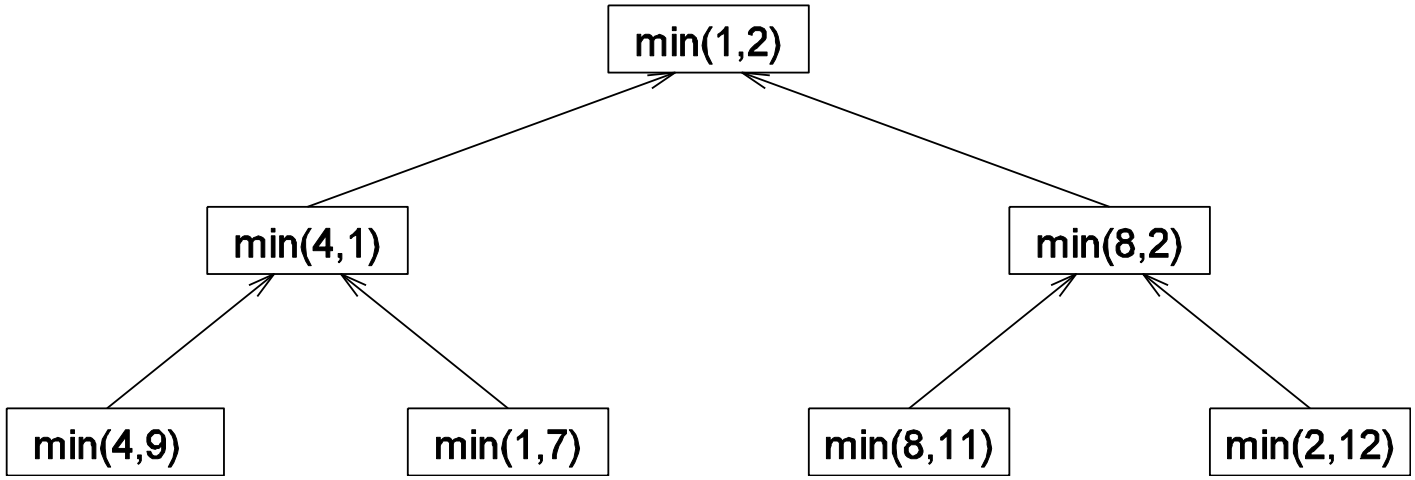
Recursive Decomposition: Example

We can rewrite the loop as follows:

```
1. procedure RECURSIVE_MIN (A, n)
2. begin
3. if ( n = 1 ) then
4.     min := A [0]  ;
5. else
6.     lmin := RECURSIVE_MIN ( A, n/2 );
7.     rmin := RECURSIVE_MIN (  &(A[n/2]), n - n/2 );
8.     if (lmin  < rmin) then
9.                 min := lmin;
10.    else
11.                min := rmin;
12.    endelse;
13. endelse;
14. return min;
15. end RECURSIVE_MIN
```

Recursive Decomposition: Example

The code in the previous foil can be decomposed naturally using a recursive decomposition strategy. We illustrate this with the following example of finding the minimum number in the set {4, 9, 1, 7, 8, 11, 2, 12}. The task dependency graph associated with this computation is as follows:



Data Decomposition

- Basic Idea: Partition data first, then infer tasks decomposition based on how computations access the data
- Approach:
  - Identify the data on which computations are performed.
  - Partition this data across various tasks.
  - This partitioning induces a decomposition of the problem.
- Data can be partitioned in various ways - this critically impacts performance of a parallel algorithm.

Data Decomposition: Output Data Decomposition

- Often, each element of the output can be computed independently of others (but simply as a function of the input).
- A partition of the output across tasks decomposes the problem naturally.

Input Data Partitioning

- Generally applicable if each output can be naturally computed as a function of the input.
- In many cases, this is the only natural decomposition because the output is not clearly known a-priori (e.g., the problem of finding the minimum in a list, sorting a given list, etc.).
- A task is associated with each input data partition. The task performs as much of the computation with its part of the data. Subsequent processing combines these partial results.

Intermediate Data Partitioning

- Computation can often be viewed as a sequence of transformation from the input to the output data.
- In these cases, it is often beneficial to use one of the intermediate stages as a basis for decomposition.

The Owner Computes Rule

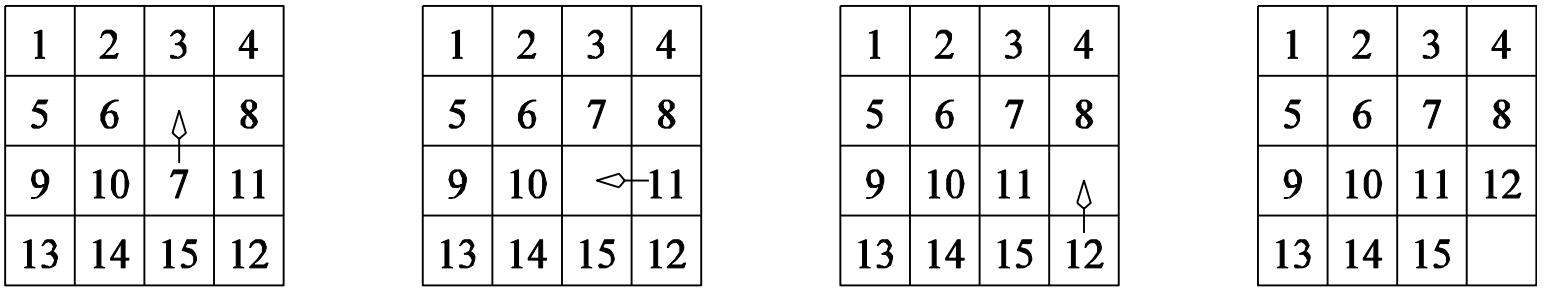
- The *Owner Computes Rule* generally states that the process assined a particular data item is responsible for all computation associated with it.
- In the case of input data decomposition, the owner computes rule imples that all computations that use the input data are performed by the process.
- In the case of output data decomposition, the owner computes rule implies that the output is computed by the process to which the output data is assigned.

Exploratory Decomposition

- In many cases, the decomposition of the problem goes hand-in-hand with its execution.
- These problems typically involve the exploration (search) of a state space of solutions.
- Problems in this class include a variety of discrete optimization problems (0/1 integer programming, QAP, etc.), theorem proving, game playing, etc.

Exploratory Decomposition: Example

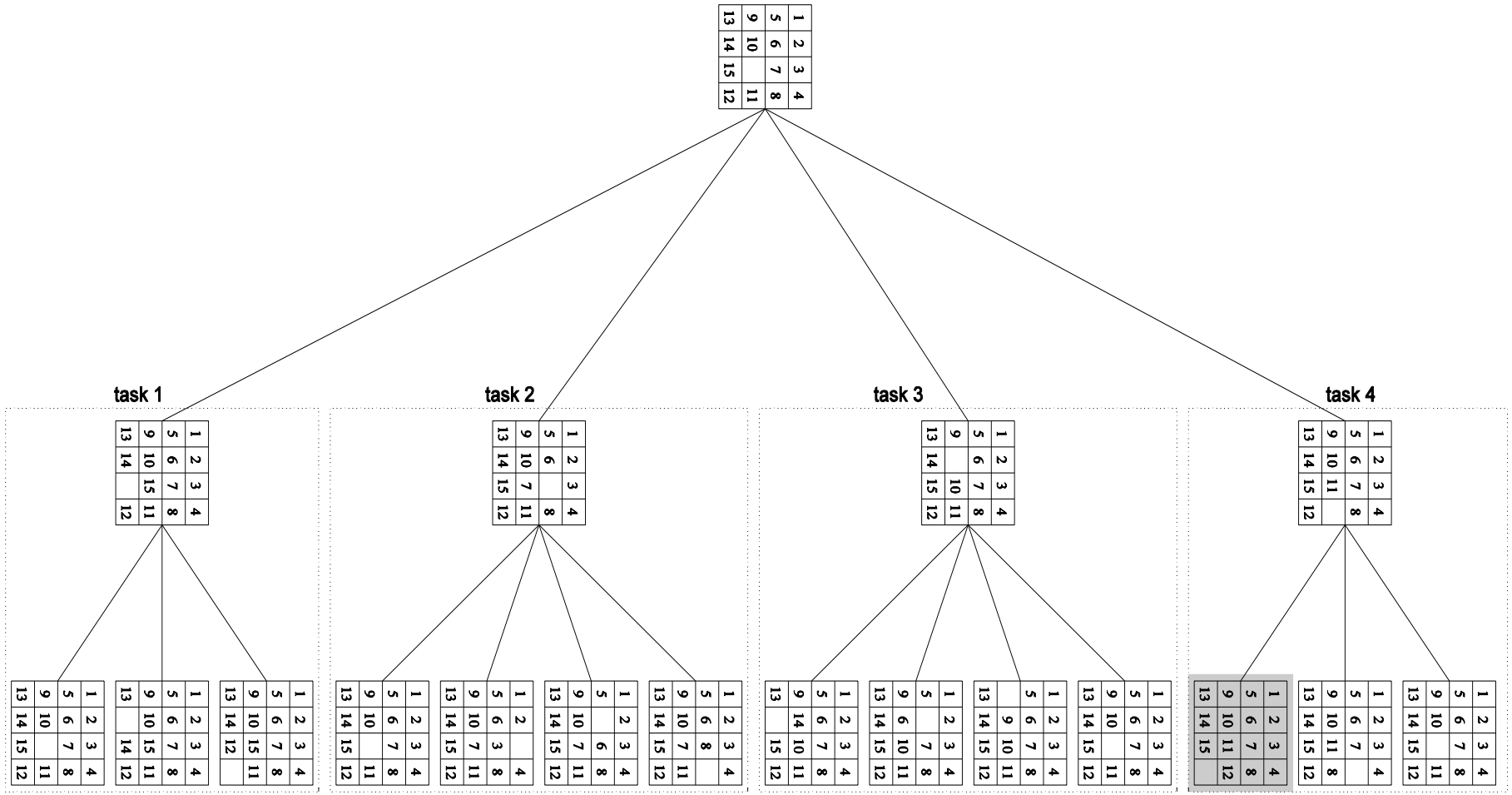
A simple application of exploratory decomposition is in the solution to a 15 puzzle (a tile puzzle). We show a sequence of three moves that transform a given initial state (a) to desired final state (d).



Of-course, the problem of computing the solution, in general, is much more difficult than in this simple example.

Exploratory Decomposition: Example

The state space can be explored by generating various successor states of the current state and to view them as independent tasks.



Speculative Decomposition

- In some applications, dependencies between tasks are not known a-priori.
- For such applications, it is impossible to identify independent tasks.
- There are generally two approaches to dealing with such applications: conservative approaches, which identify independent tasks only when they are guaranteed to not have dependencies, and, optimistic approaches, which schedule tasks even when they may potentially be erroneous.
- Conservative approaches may yield little concurrency and optimistic approaches may require roll-back mechanism in the case of an error.

Speculative Decomposition: Example

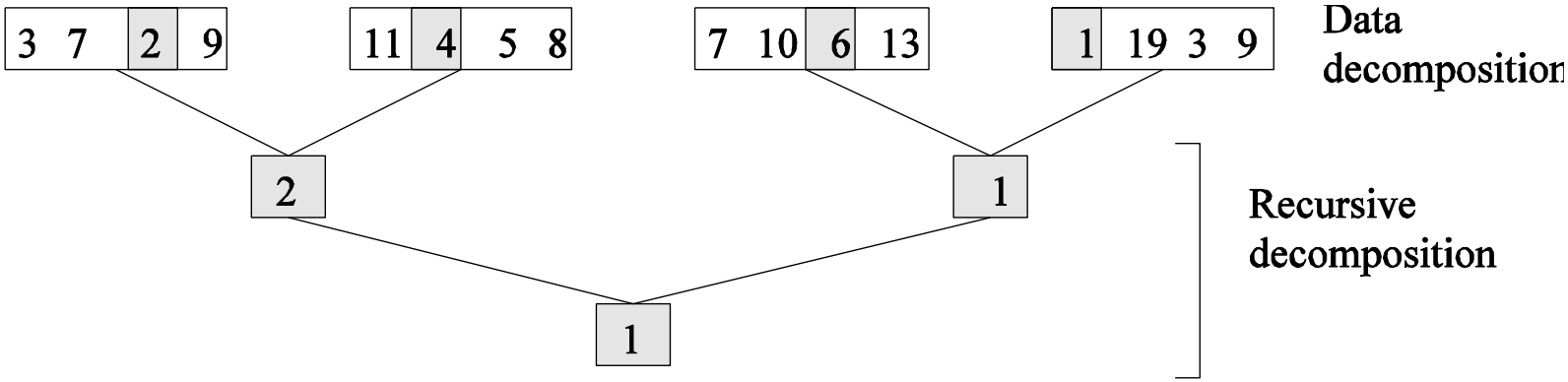
A classic example of speculative decomposition is in discrete event simulation.

- The central data structure in a discrete event simulation is a time-ordered event list.
- Events are extracted precisely in time order, processed, and if required, resulting events are inserted back into the event list.
- Consider your day today as a discrete event system - you get up, get ready, drive to work, work, eat lunch, work some more, drive back, eat dinner, and sleep.
- Each of these events may be processed independently, however, in driving to work, you might meet with an unfortunate accident and not get to work at all.
- Therefore, an optimistic scheduling of other events will have to be rolled back.

Hybrid Decompositions

Often, a mix of decomposition techniques is necessary for decomposing a problem. Consider the following examples:

- In quicksort, recursive decomposition alone limits concurrency (Why?). A mix of data and recursive decompositions is more desirable.
- In discrete event simulation, there might be concurrency in task processing. A mix of speculative decomposition and data decomposition may work well.
- Even for simple problems like finding a minimum of a list of numbers, a mix of data and recursive decomposition works well.



Mapping Techniques for Minimum Idling

Mapping techniques can be static or dynamic.

- Static Mapping: Tasks are mapped to processes a-priori. For this to work, we must have a good estimate of the size of each task. Even in these cases, the problem may be NP complete.
- Dynamic Mapping: Tasks are mapped to processes at runtime. This may be because the tasks are generated at runtime, or that their sizes are not known.

Other factors that determine the choice of techniques include the size of data associated with a task and the nature of underlyingdomain.

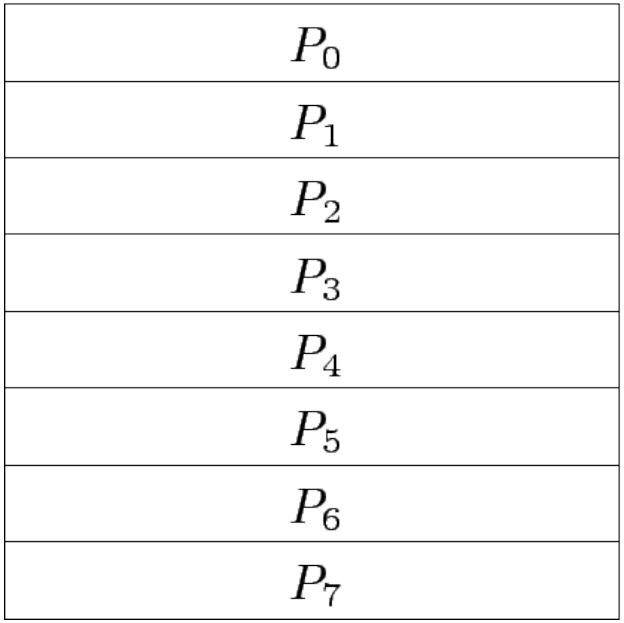
Schemes for Static Mapping

- Mappings based on data partitioning.
- Mappings based on task graph partitioning.
- Hybrid mappings.

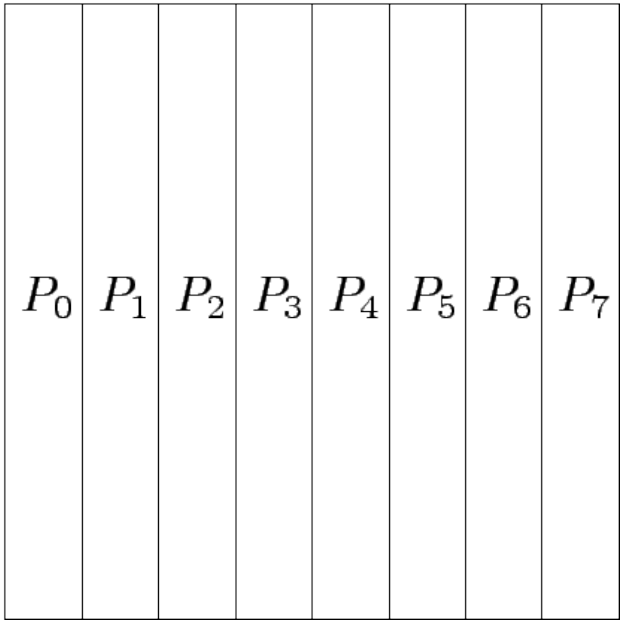
Mappings Based on Data Partitioning

We can combine data partitioning with the ``owner-computes'' rule to partition the computation into subtasks. The simplest data decomposition schemes for dense matrices are 1-D block distribution schemes.

row-wise distribution

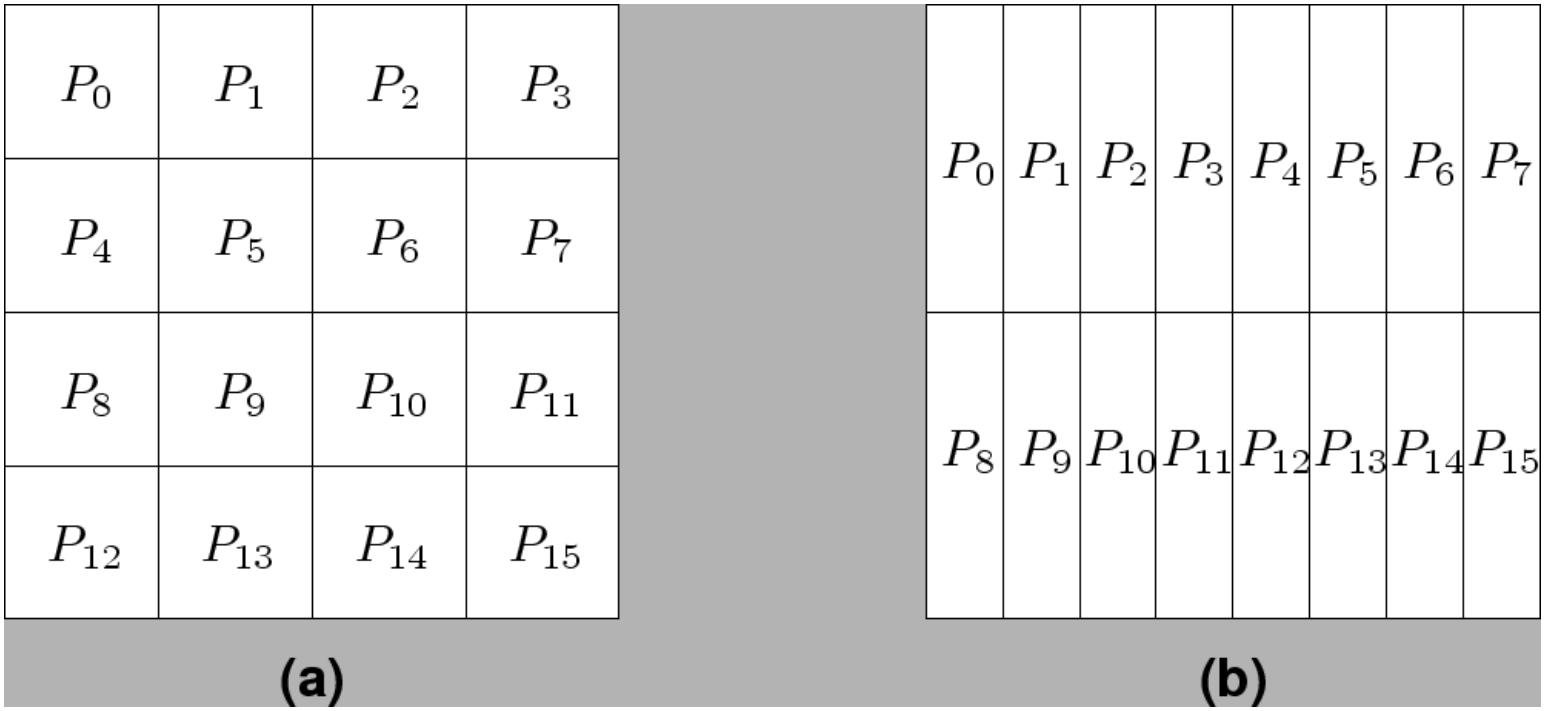


column-wise distribution



Block Array Distribution Schemes

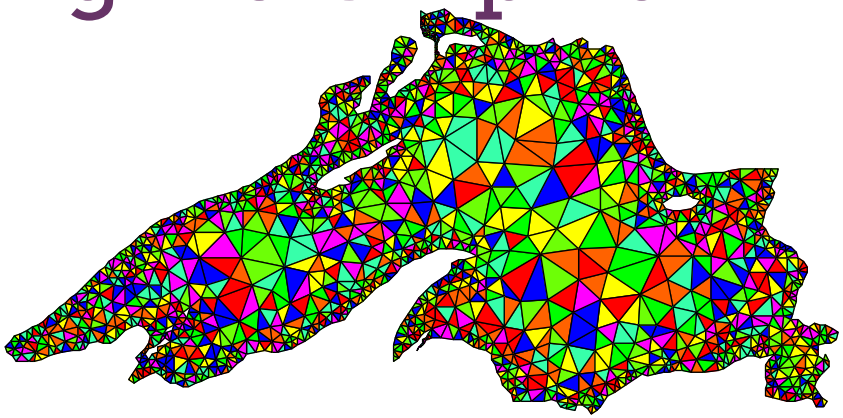
Block distribution schemes can be generalized to higher dimensions as well.



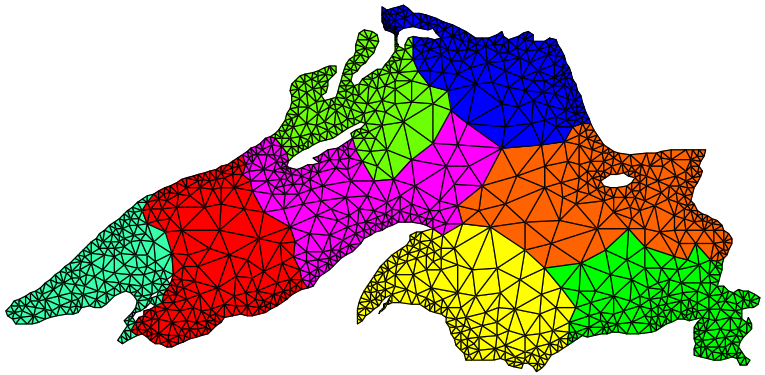
Graph Partitioning Dased Data Decomposition

- In case of sparse matrices, block decompositions are more complex.
- Consider the problem of multiplying a sparse matrix with a vector.
- The graph of the matrix is a useful indicator of the work (number of nodes) and communication (the degree of each node).
- In this case, we would like to partition the graph so as to assign equal number of nodes to each process, while minimizing edge count of the graph partition.

Partitioning the Graph of Lake Superior



Random Partitioning



Partitioning for minimum edge-cut.

Mappings Based on Task Paritioning

- Partitioning a given task-dependency graph across processes.
- Determining an optimal mapping for a general task-dependency graph is an NP-complete problem.
- Excellent heuristics exist for structured graphs.

Hierarchical Mappings

- Sometimes a single mapping technique is inadequate.
- For example, the task mapping of the binary tree (quicksort) cannot use a large number of processors.
- For this reason, task mapping can be used at the top level and data partitioning within each level.

An example of task partitioning at top level with data partitioning at the lower level.