Conducting Human-Subject Experiments with Virtual and Augmented Reality

VR 2007 Tutorial

J. Edward Swan II, Mississippi State University (organizer)
Stephen R. Ellis, NASA Ames Research Center
Bernard D. Adelstein, NASA Ames Research Center
### Schedule

<table>
<thead>
<tr>
<th>Time</th>
<th>Duration</th>
<th>Session</th>
<th>Speaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30–9:00</td>
<td>0.5 hrs</td>
<td>Intro and Group Discussion</td>
<td>All</td>
</tr>
<tr>
<td>9:00–10:00</td>
<td>1.0 hrs</td>
<td>Basic Experimental Design and Analysis</td>
<td>Ed</td>
</tr>
<tr>
<td>10:00–10:30</td>
<td>0.5 hrs</td>
<td><strong>Coffee Break</strong></td>
<td></td>
</tr>
<tr>
<td>10:30–12:00</td>
<td>1.5 hrs</td>
<td>Basic Experimental Design and Analysis</td>
<td>Ed</td>
</tr>
<tr>
<td>12:00–1:30</td>
<td>1.5 hrs</td>
<td><strong>Lunch Break</strong></td>
<td></td>
</tr>
<tr>
<td>1:30–3:00</td>
<td>1.5 hrs</td>
<td>Classical and Other Psychophysical Methods</td>
<td>Dov</td>
</tr>
<tr>
<td></td>
<td></td>
<td>for Virtual Environments</td>
<td></td>
</tr>
<tr>
<td>3:00–3:30</td>
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<td><strong>Coffee Break</strong></td>
<td></td>
</tr>
<tr>
<td>3:30–5:00</td>
<td>1.5 hrs</td>
<td>Human Performance and Preference Studies: Exhortations and Illustrations</td>
<td>Steve</td>
</tr>
</tbody>
</table>
Basic Experimental Design and Analysis

J. Edward Swan II, Ph.D.

Department of Computer Science and Engineering

Institute for Neurocognitive Science and Technology

Mississippi State University
Motivation and Goals

• Studying experimental design and analysis at Mississippi State University:
  – PSY 3103 Introduction to Psychological Statistics
  – PSY 3314 Experimental Psychology
  – PSY 6103 Psychometrics
  – PSY 8214 Quantitative Methods In Psychology II
  – PSY 8803 Advanced Quantitative Methods
  – IE 6613 Engineering Statistics I
  – IE 6623 Engineering Statistics II
  – ST 8114 Statistical Methods
  – ST 8214 Design & Analysis Of Experiments
  – ST 8853 Advanced Design of Experiments I
  – ST 8863 Advanced Design of Experiments II

• 7 undergrad hours; 30 grad hours; 3 departments!

• Course attendee backgrounds?
Motivation and Goals

• What can we accomplish in one day?

• Study subset of basic techniques
  – Presenters have found these to be the most applicable to VR, AR systems

• Focus on intuition behind basic techniques

• Become familiar with basic concepts and terms
  – Facilitate working with collaborators from psychology, industrial engineering, statistics, etc.
Outline

• **Empiricism**
• Experimental Validity
• Experimental Design
• Gathering Data
• Describing Data
  – Graphing Data
  – Descriptive Statistics
• Inferential Statistics
  – Hypothesis Testing
  – Hypothesis Testing Means
  – Power
  – Analysis of Variance and Factorial Experiments
Why Human Subject (HS) Experiments?

• VR and AR hardware / software more mature

• Focus of field:
  – Implementing technology → using technology

• Increasingly running HS experiments:
  – How do humans perceive, manipulate, cognate with VR, AR-mediated information?
  – Measure utility of VR / AR for applications

• HS experiments at VR:

<table>
<thead>
<tr>
<th>VR year</th>
<th>papers</th>
<th>%</th>
<th>sketches</th>
<th>%</th>
<th>posters</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>10 / 29</td>
<td>35%</td>
<td></td>
<td></td>
<td>5 / 14</td>
<td>36%</td>
</tr>
<tr>
<td>2004</td>
<td>9 / 26</td>
<td>35%</td>
<td></td>
<td></td>
<td>5 / 23</td>
<td>22%</td>
</tr>
<tr>
<td>2005</td>
<td>13 / 29</td>
<td>45%</td>
<td>1 / 8</td>
<td>13%</td>
<td>8 / 15</td>
<td>53%</td>
</tr>
<tr>
<td>2006</td>
<td>12 / 27</td>
<td>44%</td>
<td>2 / 10</td>
<td>20%</td>
<td>1 / 10</td>
<td>10%</td>
</tr>
<tr>
<td>2007</td>
<td>9 / 26</td>
<td>35%</td>
<td>3 / 15</td>
<td>20%</td>
<td>5 / 18</td>
<td>28%</td>
</tr>
</tbody>
</table>
Logical Deduction vs. Empiricism

• **Logical Deduction**
  – Analytic solutions in closed form
  – Amenable to proof techniques
  – Much of computer science fits here
  – Examples:
    • Computability (what can be calculated?)
    • Complexity theory (how efficient is this algorithm?)

• **Empirical Inquiry**
  – Answers questions that cannot be proved analytically
  – Much of science falls into this area
  – Antithetical to mathematics, computer science
What is Empiricism?

• The Empirical Technique
  – Develop a hypothesis, perhaps based on a theory
  – Make the hypothesis testable
  – Develop an empirical experiment
  – Collect and analyze data
  – Accept or refute the hypothesis
  – Relate the results back to the theory
  – If worthy, communicate the results to your community

• Statistics:
  – Foundation for empirical work; necessary but not sufficient
  – Often not useful for managing problems of gathering, interpreting, and communicating empirical information.
Where is Empiricism Used?

• Humans are very non-analytic

• Fields that study humans:
  – Psychology / social sciences
  – Industrial engineering
  – Ergonomics
  – Business / management
  – Medicine

• Fields that don’t study humans:
  – Agriculture, natural sciences, etc.

• Computer Science:
  – HCI
  – Software engineering
Experimental Validity

• Empiricism
• *Experimental Validity*
• Experimental Design
• Gathering Data
• Describing Data
  – Graphing Data
  – Descriptive Statistics
• Inferential Statistics
  – Hypothesis Testing
  – Hypothesis Testing Means
  – Power
  – Analysis of Variance and Factorial Experiments
Designing Valid Empirical Experiments

• Experimental Validity
  – Does experiment really measure what we want it to measure?
  – Do our results really mean what we think (and hope) they mean?
  – Are our results reliable?
    • If we run the experiment again, will we get the same results?
    • Will others get the same results?

• Validity is a large topic in empirical inquiry
Experimental Variables

• Independent Variables
  – What the experiment is studying
  – Occur at different levels
    • Example: stereopsis, at the levels of stereo, mono
  – Systematically varied by experiment

• Dependent Variables
  – What the experiment measures
  – Assume dependent variables will be effected by independent variables
  – Must be measurable quantities
    • Time, task completion counts, error counts, survey answers, scores, etc.
    • Example: VR navigation performance, in total time
Experimental Variables

• Independent variables can vary in two ways
  – **Between-subjects**: each subject sees a different level of the variable
    • Example: ½ of subjects see stereo, ½ see mono
  – **Within-subjects**: each subject sees all levels of the variable
    • Example: each subject sees both stereo and mono

• **Confounding factors** (or confounding variables)
  – Factors that are not being studied, but will still affect experiment
    • Example: stereo condition less bright than mono condition
  – Important to **predict and control confounding factors**, or experimental validity will suffer
Experimental Design

- Empiricism
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- *Experimental Design*
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Experimental Designs

• 2 x 1 is simplest possible design, with one independent variable at two levels:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Stereopsis</th>
</tr>
</thead>
<tbody>
<tr>
<td>level 1</td>
<td>stereo</td>
</tr>
<tr>
<td>level 2</td>
<td>mono</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subjects</th>
<th>1st condition</th>
<th>2nd condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 3, 5, 7</td>
<td>stereo</td>
<td>mono</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>mono</td>
<td>stereo</td>
</tr>
</tbody>
</table>

• Important confounding factors for within subject variables:
  – Learning effects
  – Fatigue effects
• Control these by **counterbalancing** the design
  – Ensure no systematic variation between levels and the order they are presented to subjects
Factorial Designs

- \( n \times 1 \) designs generalize the number of levels:

<table>
<thead>
<tr>
<th>VE terrain type</th>
</tr>
</thead>
<tbody>
<tr>
<td>flat</td>
</tr>
<tr>
<td>hilly</td>
</tr>
<tr>
<td>mountainous</td>
</tr>
</tbody>
</table>

- **Factorial designs** generalize number of independent variables and the number of levels of each variable
- Examples: \( n \times m \) design, \( n \times m \times p \) design, etc.
- Must watch for factorial explosion of design size!

3 x 2 design:

<table>
<thead>
<tr>
<th>VE terrain type</th>
<th>Stereopsis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>stereo</td>
</tr>
<tr>
<td>flat</td>
<td></td>
</tr>
<tr>
<td>hilly</td>
<td></td>
</tr>
<tr>
<td>mountainous</td>
<td></td>
</tr>
</tbody>
</table>
Cells and Levels

- **Cell**: each combination of levels
- **Repetitions**: typically, the combination of levels at each cell is repeated a number of times

Example of how this design might be described:
- “A 3 (VE terrain type) by 2 (stereopsis) within-subjects design, with 4 repetitions of each cell.”
- This means each subject would see $3 \times 2 \times 4 = 24$ total conditions
- The presentation order would be counterbalanced
Counterbalancing

• Addresses time-based confounding factors:
  – Within-subjects variables: control learning and fatigue effects
  – Between-subjects variables: control calibration drift, weather, other factors that vary with time

• There are two counterbalancing methods:
  – Random permutations
  – Systematic variation
    • Latin squares are a very useful and popular technique

\[
\begin{bmatrix}
1 & 2 \\
2 & 1 \\
\end{bmatrix} \quad \begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 1 \\
3 & 1 & 2 \\
1 & 3 & 2 \\
2 & 1 & 3 \\
3 & 2 & 1 \\
\end{bmatrix} \quad \begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 4 & 1 & 3 \\
3 & 1 & 4 & 2 \\
4 & 3 & 2 & 1 \\
\end{bmatrix}
\]

- Latin square properties:
  – Every level appears in every position the same number of times
  – Every level is followed by every other level
  – Every level is preceded by every other level

\[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 3 & 1 \\
3 & 1 & 2 \\
1 & 3 & 2 \\
2 & 1 & 3 \\
3 & 2 & 1 \\
\end{bmatrix} \quad \begin{bmatrix}
1 & 2 & 3 & 4 \\
2 & 4 & 1 & 3 \\
3 & 1 & 4 & 2 \\
4 & 3 & 2 & 1 \\
\end{bmatrix}
\]

\(2 \times 2\) \quad \(6 \times 3\) (there is no \(3 \times 3\) that has all 3 properties)
Counterbalancing Example

- “A 3 (VE terrain type) by 2 (stereopsis) within-subjects design, with 4 repetitions of each cell.”
- Form Cartesian product of Latin squares
  {6 x 3} (VE Terrain Type) \(\otimes\) {2 x 2} (Stereopsis)
- Perfectly counterbalances groups of 12 subjects

<table>
<thead>
<tr>
<th>Subject</th>
<th>Presentation Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1A, 1B, 2A, 2B, 3A, 3B</td>
</tr>
<tr>
<td>2</td>
<td>1B, 1A, 2B, 2A, 3B, 3A</td>
</tr>
<tr>
<td>3</td>
<td>2A, 2B, 3A, 3B, 1A, 1B</td>
</tr>
<tr>
<td>4</td>
<td>2B, 2A, 3B, 3A, 1B, 1A</td>
</tr>
<tr>
<td>5</td>
<td>3A, 3B, 1A, 1B, 2A, 2B</td>
</tr>
<tr>
<td>6</td>
<td>3B, 3A, 1B, 1A, 2B, 2A</td>
</tr>
<tr>
<td>7</td>
<td>1A, 1B, 3A, 3B, 2A, 2B</td>
</tr>
<tr>
<td>8</td>
<td>1B, 1A, 3B, 3A, 2B, 2A</td>
</tr>
<tr>
<td>9</td>
<td>2A, 2B, 1A, 1B, 3A, 3B</td>
</tr>
<tr>
<td>10</td>
<td>2B, 2A, 1B, 1A, 3B, 3A</td>
</tr>
<tr>
<td>11</td>
<td>3A, 3B, 2A, 2B, 1A, 1B</td>
</tr>
<tr>
<td>12</td>
<td>3B, 3A, 2B, 2A, 1B, 1A</td>
</tr>
</tbody>
</table>
Experimental Design Example #1

All variables within-subject

From [Living et al. 03]
# Experimental Design Example #2

<table>
<thead>
<tr>
<th>Stereo Viewing</th>
<th>on</th>
<th>off</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Movement</td>
<td>rate</td>
<td>position</td>
</tr>
<tr>
<td>Frame of Reference</td>
<td>ego</td>
<td>exo</td>
</tr>
<tr>
<td></td>
<td>Wall</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Workbench</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Desktop</td>
<td></td>
</tr>
</tbody>
</table>

- Mixed design: some variables between-subject, others within-subject.

From [Swan et al. 03]
Gathering Data

- Empiricism
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Gathering Data

• Some unique aspects of VR and AR
  – Can capture, log, and analyze tracker trajectory
  – If we log head / hand trajectory so we can play it back, must have way of logging critical incidents
  – VR / AR equipment more fragile than other UI setups

  – In a CAVE:
    • Observing a subject can break their presence / immersion
    • Determining button presses when experimenter cannot see wand

  – In AR, very difficult to know what user is seeing
    • Can mount separate display near user or on their back
    • Could mount lightweight camera on user’s head

• Measurable phenomena:
  – Button presses, physical actions, answers
Pilot Testing a Design

• Experimental designs have to be tested and iterated (debugged)

• Typical flow:
  – 1st run: subjects are you, collaborators
  – 2nd run: small number of preliminary subjects
  – 3rd run: subset of real subjects

• With each run, problems are revealed; fix and iterate

• For later runs, perform data analysis before gathering additional data
Graphing Data

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Types of Statistics

• Descriptive Statistics
  – Describe and explore data
  – Summary statistics:
    many numbers → few numbers
  – All types of graphs and visual representations
  – Data analysis begins with descriptive stats
    • Understand data distribution
    • Test assumptions of significance tests

• Inferential Statistics
  – Detect relationships in data
  – Significance tests
  – Infer population characteristics from sample characteristics
Exploring Data with Graphs

- Histogram common data overview method

median = 59.5  mean = 60.26  mode = 62

From [Howell 02] p 21
Classifying Data with Histograms

From [Howell 02] p 28
Stem-and-Leaf:
Histogram From Actual Data

From [Howell 02] p 21, 23

From [Howell 02] p 21, 23
Stem-and-Leaf:
Histogram From Actual Data

Final Recorded Grades

1  3% F  0  0
0  0% F  1
0  0% F  2
0  0% F  3
0  0% F  4
0  0% F  5
5  16% D  6  34788
8  26% C  7  12233469
8  26% B  8  01244699
9  29% A  9  001123346
31

Grades from my Autumn 2005 analysis of algorithms class
Boxplot

• Emphasizes variation and relationship to mean
• Because narrow, can be used to display side-by-side groups

Data from [Swan et al. 06]
Example Histogram and Boxplot from Real Data

- **mean = 2355**
- **median = 1453**
- **min value**
- **25th**
- **75th**
- **upper fence**
- **max values (outliers)**

Data from [Living et al. 03]
We Have Only Scratched the Surface…

- There are a very large number of graphing techniques
- Tufte’s [83, 90] works are classic, and stat books show many more examples (e.g. Howell [03]).

Lots of good examples…

And plenty of bad examples!

From [Tufte 83], p 134, 62
Descriptive Statistics

• Empiricism
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• Usability Engineering
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Summary Statistics

• Many numbers $\rightarrow$ few numbers

• Measures of central tendency:
  – Mean: average
  – Median: middle data value
  – Mode: most common data value

• Measures of variability / dispersion:
  – Mean absolute deviation
  – Variance
  – Standard Deviation
Populations and Samples

• Population:
  – Set containing every possible element that we want to measure
  – Usually a Platonic, theoretical construct
  – Mean: $\mu$  Variance: $\sigma^2$  Standard deviation: $\sigma$

• Sample:
  – Set containing the elements we actually measure (our subjects)
  – Subset of related population
  – Mean: $\bar{X}$  Variance: $s^2$  Standard deviation: $s$
  Number of samples: $N$
Measuring Variability / Dispersion

Mean:
\[ \bar{X} = \frac{\sum X}{N} \]

Variance:
\[ s^2 = \frac{\sum (X - \bar{X})^2}{N - 1} \]

\[ \sigma^2 = \frac{\sum (X - \mu)^2}{N} \]

Mean absolute deviation:
\[ \text{m.a.d.} = \frac{\sum |X - \bar{X}|}{N} \]

Standard deviation:
\[ s = \sqrt{\frac{\sum (X - \bar{X})^2}{N - 1}} \]

• Standard deviation uses same units as samples and mean.
• Calculation of population variance \( \sigma^2 \) is theoretical, because \( \mu \) almost never known and the population size \( N \) would be very large (perhaps infinity).
Sums of Squares, Degrees of Freedom, Mean Squares

• Very common terms and concepts

\[
s^2 = \frac{\sum(X - \bar{X})^2}{N - 1} = \frac{SS}{df} = \frac{\text{sums of squares}}{\text{degrees of freedom}} = \text{MS (mean squares)}
\]

• Sums of squares:
  – Summed squared deviations from mean

• Degrees of freedom:
  – Given a set of \( N \) observations used in a calculation, how many numbers in the set may vary
  – Equal to \( N \) minus number of means calculated

• Mean squares:
  – Sums of squares divided by degrees of freedom
  – Another term for variance, used in ANOVA
Hypothesis Testing

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• **Inferential Statistics**
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Hypothesis Testing

• Goal is to infer population characteristics from sample characteristics

From [Howell 02], p 78
Testable Hypothesis

• **General hypothesis:** The research question that motivates the experiment.

• **Testable hypothesis:** The research question expressed in a way that can be measured and studied.

• Generating a **good** testable hypothesis is a real skill of experimental design.
  – By *good*, we mean contributes to experimental validity.
  – Skill best learned by studying and critiquing previous experiments.
Testable Hypothesis Example

- **General hypothesis:** Stereo will make people more effective when navigating through a virtual environment (VE).

- **Testable hypothesis:** We measure time it takes for subjects to navigate through a particular VE, under conditions of stereo and mono viewing. We hypothesis subjects will be faster under stereo viewing.

- Testable hypothesis requires a measurable quantity:
  - Time, task completion counts, error counts, etc.

- Some factors effecting experimental validity:
  - Is VE representative of something interesting (e.g., a real-world situation)?
  - Is navigation task representative of something interesting?
  - Is there an underlying theory of human performance that can help predict the results? Could our results contribute to this theory?
What Are the Possible Alternatives?

• Let time to navigate be $\mu_s$: stereo time; $\mu_m$: mono time
  – Perhaps there are two populations: $\mu_s - \mu_m = d$
    (they could be close together)
  – Perhaps there is one population: $\mu_s - \mu_m = 0$
    (they could be far apart)
Hypothesis Testing Procedure

1. Develop testable hypothesis \( H_1: \mu_s - \mu_m = d \)
   - (E.g., subjects faster under stereo viewing)

2. Develop null hypothesis \( H_0: \mu_s - \mu_m = 0 \)
   - Logical opposite of testable hypothesis

3. Construct sampling distribution assuming \( H_0 \) is true.

4. Run an experiment and collect samples; yielding sampling statistic \( X \).
   - (E.g., measure subjects under stereo and mono conditions)

5. Referring to sampling distribution, calculate conditional probability of seeing \( X \) given \( H_0: p( X \mid H_0 ). \)
   - If probability is low (\( p \leq 0.05, p \leq 0.01 \)), we are unlikely to see \( X \) when \( H_0 \) is true. We reject \( H_0 \), and embrace \( H_1 \).
   - If probability is not low (\( p > 0.05 \)), we are likely to see \( X \) when \( H_0 \) is true. We do not reject \( H_0 \).
Example 1: VE Navigation with Stereo Viewing

1. Hypothesis $H_1: \mu_s - \mu_m = d$
   - Subjects faster under stereo viewing.

2. Null hypothesis $H_0: \mu_s - \mu_m = 0$
   - Subjects same speed whether stereo or mono viewing.

3. Constructed sampling distribution assuming $H_0$ is true.

4. Ran an experiment and collected samples:
   - 32 subjects, collected 128 samples
   - $X_s = 36.431$ sec; $X_m = 34.449$ sec; $X_s - X_m = 1.983$ sec

5. Calculated conditional probability of seeing 1.983 sec given $H_0$: $p(1.983$ sec $| H_0) = 0.445$.
   - $p = 0.445$ not low, we are likely to see 1.983 sec when $H_0$ is true. We do not reject $H_0$.
   - This experiment did not tell us that subjects were faster under stereo viewing.

Data from [Swan et al. 03]
Example 2: Effect of Intensity on AR Occluded Layer Perception

1. Hypothesis $H_1$: $\mu_c - \mu_d = d$
   – Tested constant and decreasing intensity. Subjects faster under decreasing intensity.

2. Null hypothesis $H_0$: $\mu_c - \mu_d = 0$
   – Subjects same speed whether constant or decreasing intensity.

3. Constructed sampling distribution assuming $H_0$ is true.

4. Ran an experiment and collected samples:
   – 8 subjects, collected 1728 samples
   – $X_c = 2592.4$ msec; $X_d = 2339.9$ msec; $X_c - X_d = 252.5$ msec

5. Calculated conditional probability of seeing 252.5 msec given $H_0$: $p(252.5 \text{ msec} \mid H_0) = 0.008$.
   – $p = 0.008$ is low ($p \leq 0.01$); we are unlikely to see 252.5 msec when $H_0$ is true. We reject $H_0$, and embrace $H_1$.
   – This experiment suggests that subjects are faster under decreasing intensity.

Data from [Living et al. 03]
Some Considerations...

• The conditional probability $p(X | H_0)$
  – Much of statistics involves how to calculate this probability; source of most of statistic’s complexity
  – Logic of hypothesis testing the same regardless of how $p(X | H_0)$ is calculated
  – If you can calculate $p(X | H_0)$, you can test a hypothesis

• The null hypothesis $H_0$
  – $H_0$ usually in form $f(\mu_1, \mu_2, \ldots) = 0$
  – Gives hypothesis testing a double-negative logic: assume $H_0$ as the opposite of $H_1$, then reject $H_0$
  – Philosophy is that can never prove something true, but can prove it false
  – $H_1$ usually in form $f(\mu_1, \mu_2, \ldots) \neq 0$; we don’t know what value it will take, but main interest is that it is not 0
When We Reject $H_0$

• Calculate $\alpha = P(X | H_0)$, when do we reject $H_0$?
  – In psychology, two levels: $\alpha \leq 0.05; \alpha \leq 0.01$
  – Other fields have different values

• What can we say when we reject $H_0$ at $\alpha = 0.008$?
  – “If $H_0$ is true, there is only an 0.008 probability of getting our results, and this is unlikely.”
    • Correct!
  
  – “There is only a 0.008 probability that our result is in error.”
    • Wrong, this statement refers to $P(H_0)$, but that’s not what we calculated.
  
  – “There is only a 0.008 probability that $H_0$ could have been true in this experiment.”
    • Wrong, this statement refers to $P(H_0 | X)$, but that’s not what we calculated.
When We Don’t Reject $H_0$

• What can we say when we don’t reject $H_0$ at $\alpha = 0.445$?
  – “We have proved that $H_0$ is true.”
  – “Our experiment indicates that $H_0$ is true.”
    • Wrong, statisticians agree that hypothesis testing cannot prove $H_0$ is true.

• Statisticians do not agree on what failing to reject $H_0$ means.
  – Conservative viewpoint (Fisher):
    • We must suspend judgment, and cannot say anything about the truth of $H_0$.
  – Alternative viewpoint (Neyman & Pearson):
    • We “accept” $H_0$, and act as if it’s true for now…
    • But future data may cause us to change our mind

From [Howell 02], p 99
Probabilistic Reasoning

• If hypothesis testing was absolute:
  – If $H_0$ is true, then $X$ cannot occur...however, $X$ has occurred...therefore $H_0$ is false.
  – e.g.: If a person is a Martian, then they are not a member of Congress (true)...this person is a member of Congress...therefore they are not a Martian. (correct result)
  – e.g.: If a person is an American, then they are not a member of Congress (false)...this person is a member of Congress...therefore they are not an American. (correct result because if-then false)

• However, hypothesis testing is probabilistic:
  – If $H_0$ is true, then $X$ is highly unlikely...however, $X$ has occurred...therefore $H_0$ is highly unlikely.
  – e.g.: If a person is an American, then they are probably not a member of Congress (true, right?)...this person is a member of Congress...therefore they are probably not an American. (correct hypothesis testing reasoning, but incorrect result)

From [Cohen 94]
Hypothesis Testing Outcomes

<table>
<thead>
<tr>
<th>Decision</th>
<th>Reject $H_0$</th>
<th>Don’t reject $H_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True state of the world</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_0$ false</td>
<td>correct a result!</td>
<td>wrong type II error</td>
</tr>
<tr>
<td></td>
<td>$p = 1 - \beta = \text{power}$</td>
<td>$p = \beta$</td>
</tr>
<tr>
<td>$H_0$ true</td>
<td>wrong type I error</td>
<td>correct (but wasted time)</td>
</tr>
<tr>
<td></td>
<td>$p = \alpha$</td>
<td>$p = 1 - \alpha$</td>
</tr>
</tbody>
</table>

- $p(X \mid H_0)$ compared to $\alpha$, so hypothesis testing involves setting $\alpha$ (typically 0.05 or 0.01)
- Two ways to be right:
  - Find a result
  - Fail to find a result and waste time running an experiment
- Two ways to be wrong:
  - Type I error: we think we have a result, but we are wrong
  - Type II error: a result was there, but we missed it
When Do We Really Believe a Result?

- When we reject $H_0$, we have a result, but:
  - It’s possible we made a type I error
  - It’s possible our finding is not reliable
    - Just an artifact of our particular experiment

- So when do we really believe a result?
  - Statistical evidence
    - $\alpha$ level: ($p < .05$, $p < .01$, $p < .001$)
    - Power
  - Meta-statistical evidence
    - Plausible explanation of observed phenomena
      - Based on theories of human behavior: perceptual, cognitive psychology; control theory, etc.
    - Repeated results
      - Especially by others
Hypothesis Testing Means

- Empiricism
- Experimental Validity
- Experimental Design
- Gathering Data
- Describing Data
  - Graphing Data
  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - *Hypothesis Testing Means*
  - Power
  - Analysis of Variance and Factorial Experiments
Hypothesis Testing Means

• How do we calculate $\alpha = \rho( X \mid H_0 )$, when $X$ is a mean?
  – Calculation possible for other statistics, but most common for means

• Answer: we refer to a sampling distribution

• We have two conceptual functions:
  – Population: unknowable property of the universe
  – Distribution: analytically defined function, has been found to match certain population statistics
Calculating $\alpha = p( X | H_0 )$ with A Sampling Distribution

• Sampling distributions are analytic functions with area 1
• To calculate $\alpha = p( X | H_0 )$ given a distribution, we first calculate the value $D$, which comes from an equation of the form:

$$D = \left( \frac{\text{size of effect} : f(\overline{X})}{\text{variability of effect} : f(s^2, N)} \right)$$

• $\alpha = p( X | H_0 )$ is equal to:
  - Probability of seeing a value $\geq |D|$
  - $2 \ast (\text{area of the distribution to the right of } |D|)$
• If $H_0$ true, we expect $D$ to be near central peek of distribution
• If $D$ far from central peek, we have reason to reject the idea that $H_0$ is true

Represents assumption that $H_0$ true
A Distribution for Hypothesis Testing Means

• The Standard Normal Distribution ($\mu = 0$, $\sigma = 1$) (also called the Z-distribution):

$$N(X; \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$$
The Central Limit Theorem

• Full Statement:
  – Given population with \((\mu, \sigma^2)\), the sampling distribution of means drawn from this population is distributed \((\mu, \sigma^2/n)\), where \(n\) is the sample size. As \(n\) increases, the sampling distribution of means approaches the normal distribution.

• Implication:
  – As \(n\) increases, distribution of means becomes normal, regardless of how “non-normal” the population looks.

• How big does \(n\) have to be before means look normally distributed?
  – For very “non-normal” data, \(n \approx 30\).
Central Limit Theorem in Action

Response time data set $A$; $N = 3436$ data points. Data from [Living et al. 03].

Plotting 100 means drawn from $A$ at random without replacement, where $n$ is number of samples used to calculate mean.

- This demonstrates:
  - As number of samples increases, distribution of means approaches normal distribution;
  - Regardless of how “non-normal” the source distribution is!
The $t$ Distribution

- In practice, when $H_0: \mu_c - \mu_d = 0$ (two means come from same population), we calculate $\alpha = \rho( X | H_0 )$ from $t$ distribution, not $Z$ distribution.
- Why? $Z$ requires the population parameter $\sigma^2$, but $\sigma^2$ almost never known. We estimate $\sigma^2$ with $s^2$, but $s^2$ biased to underestimate $\sigma^2$. Thus, $t$ more spread out than $Z$ distribution.
- $t$ distribution parametric: parameter is $df$ (degrees of freedom).

From [Howell 02], p 185
t-Test Example

• Null hypothesis $H_0$: $\mu_s - \mu_m = 0$
  – Subjects same speed whether stereo or mono viewing.

• Ran an experiment and collected samples:
  – 32 subjects, collected 128 samples
  – $n_s = 64$, $X_s = 36.431$ sec, $s_s = 15.954$ sec
  – $n_m = 64$, $X_m = 34.449$ sec, $s_m = 13.175$ sec

$$t(126) = \frac{f(\overline{X})}{f(s^2, N)} = \frac{\overline{X}_s - \overline{X}_m}{\sqrt{s_p^2\left(\frac{1}{n_s} + \frac{1}{n_m}\right)}} = 0.766, s_p^2 = \frac{(n_s - 1)s_s^2 + (n_m - 1)s_m^2}{n_s + n_m - 2}$$

• Look up $t(126) = 0.766$ in a $t$-distribution table: 0.445

• Thus, $\alpha = p(1.983 \text{ sec} \mid H_0) = 0.445$, and we do not reject $H_0$.

Calculation described by [Howell 02], p 202
One- and Two-Tailed Tests

- **t-Test example is a two-tailed test.**
  - Testing whether two means differ, no preferred direction of difference: $H_1: \mu_s - \mu_m = d$, either $\mu_s > \mu_m$ or $\mu_s < \mu_m$
  - E.g. comparing stereo or mono in VE: either might be faster
  - Most stat packages return two-tailed results by default

- **One-tailed test** is performed when preferred direction of difference: $H_1: \mu_s > \mu_m$
  - E.g. in [Meehan et al. 03], hypothesis is that heart rate & skin conductance will rise in stressful virtual environment
Power

• Empiricism
• Experimental Validity
• Experimental Design
• Gathering Data
• Describing Data
  – Graphing Data
  – Descriptive Statistics
• Inferential Statistics
  – Hypothesis Testing
  – Hypothesis Testing Means
  – Power
  – Analysis of Variance and Factorial Experiments
Interpreting $\alpha$, $\beta$, and Power

If $H_0$ is true:
- $\alpha$ is probability we make a type I error: we think we have a result, but we are wrong

If $H_1$ is true:
- $\beta$ is probability we make a type II error: a result was there, but we missed it
- Power is a more common term than $\beta$

<table>
<thead>
<tr>
<th>True state of the world</th>
<th>Decision</th>
<th>$H_0$ false</th>
<th>$H_0$ true</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$</td>
<td>a result!</td>
<td>$p = 1 - \beta = \text{power}$</td>
<td>type I error</td>
</tr>
<tr>
<td>Don't reject $H_0$</td>
<td>type II error</td>
<td>$p = \beta$</td>
<td>wasted time</td>
</tr>
</tbody>
</table>

power = $1 - \beta$
Increasing Power by Increasing $\alpha$

- Illustrates $\alpha$ / power tradeoff

- Increasing $\alpha$:
  - Increases power
  - Decreases type II error
  - Increases type I error

- Decreasing $\alpha$:
  - Decreases power
  - Increases type II error
  - Decreases type I error
Increasing Power by Measuring a Bigger Effect

• If the effect size is large:
  – Power increases
  – Type II error decreases
  – $\alpha$ and type I error stay the same

• Unsurprisingly, large effects are easier to detect than small effects
Increasing Power by Collecting More Data

- Increasing sample size ($N$):
  - Decreases variance
  - Increases power
  - Decreases **type II error**
  - $\alpha$ and **type I error** stay the same

- There are techniques that give the value of $N$ required for a certain power level.

- Here, effect size remains the same, but variance drops by half.
Using Power

• Need $\alpha$, effect size, and sample size for power:
  \[
  \text{power} = f(\,\alpha, \,|\mu_0 - \mu_1|, \,N)\]

• Problem for VR / AR:
  – Effect size $|\mu_0 - \mu_1|$ hard to know in our field
    • Population parameters estimated from prior studies
    • But our field is so new, not many prior studies
  – Can find effect sizes in more mature fields

• Post-hoc power analysis:
  \[
  \text{effect size} = |X_0 - X_1|\]
  – Estimate from sample statistics
  – But this makes statisticians grumble
    (e.g. [Howell 02] [Cohen 88])
Other Uses for Power

1. Number samples needed for certain power level:
   \[ N = f(\text{power, } \alpha, |\mu_0 - \mu_1| \text{ or } |X_0 - X_1| ) \]
   - Number extra samples needed for more powerful result
   - Gives “rational basis” for deciding \( N \) [Cohen 88]

2. Effect size that will be detectable:
   \[ |\mu_0 - \mu_1| = f( N, \text{power, } \alpha ) \]

3. Significance level needed:
   \[ \alpha = f( |\mu_0 - \mu_1| \text{ or } |X_0 - X_1|, N, \text{power} ) \]

(1) is the most common power usage
Arguing the Null Hypothesis

- Cannot directly argue $H_0: \mu_s - \mu_m = 0$. But we can argue that $|\mu_0 - \mu_1| < d$.
  - Thus, we have bound our effect size by $d$.
  - If $d$ is small, effectively argued null hypothesis.

From [Cohen 88], p 16
Example of Arguing $H_0$

- We know GP is effective depth cue, but can we get close with other graphical cues?

<table>
<thead>
<tr>
<th>ground plane</th>
<th>drawing style</th>
<th>opacity</th>
<th>intensity</th>
<th>mean error*</th>
</tr>
</thead>
<tbody>
<tr>
<td>on</td>
<td>all levels</td>
<td>both levels</td>
<td>both levels</td>
<td>0.144</td>
</tr>
<tr>
<td>off</td>
<td>wire+fill</td>
<td>decreasing</td>
<td>decreasing</td>
<td>0.111</td>
</tr>
</tbody>
</table>

* $F(1,1870) = 1.002, p = .317$

- Our effect size is $d = .087$ standard deviations
  
  \[
  \text{power}( \alpha = .05, d = 0.087, N = 265 ) = \frac{1}{17}
  \]

- Not very powerful. Where can our experiment bound $d$?
  \[d( N = 265, \text{power} = .95, \alpha = .05 ) = .31 \text{ standard deviations}\]

- This bound is significant at $\alpha = .05$, $\beta = .05$, using same logic as hypothesis testing. But how meaningful is $d < .31$? Other significant $d$’s:
  \[.37, .12, .093, .19\]

- Not very meaningful. If we ran an experiment to bound $d < .1$, how much data would we need?
  \[N( \text{power} = .95, \alpha = .05, d = .1 ) = 2600\]

- Original study collected $N = 3456$, so $N = 2600$ reasonable
Analysis of Variance and Factorial Experiments

- Empiricism
- Experimental Validity
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  - Descriptive Statistics
- Inferential Statistics
  - Hypothesis Testing
  - Hypothesis Testing Means
  - Power
  - *Analysis of Variance and Factorial Experiments*
ANOVA: Analysis of Variance

• *t*-test used for comparing two means
  – (2 x 1 designs)

• ANOVA used for factorial designs
  – Comparing multiple levels (n x 1 designs)
  – Comparing multiple independent variables (n x m, n x m x p), etc.
  – Can also compare two levels (2 x 1 designs);
    ANOVA can be considered a generalization of a *t*-Test

• No limit to experimental design size or complexity

• Most widely used statistical test in psychological research

• ANOVA based on the *F* Distribution;
  also called an *F*-Test
How ANOVA Works

- Null hypothesis $H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; $H_1$: at least one mean differs
- Estimate variance between each group: $MS_{\text{between}}$
  - Based on the difference between group means
  - If $H_0$ is true, accurate estimation
  - If $H_0$ is false, biased estimation: overestimates variance
- Estimate variance within each group: $MS_{\text{within}}$
  - Treats each group separately
  - Accurate estimation whether $H_0$ is true or false
- Calculate $F$ critical value from ratio: $F = \frac{MS_{\text{between}}}{MS_{\text{within}}}$
  - If $F \approx 1$, then accept $H_0$
  - If $F >> 1$, then reject $H_0$
ANOVA Uses The $F$ Distribution

- Calculate $\alpha = \rho(X | H_0)$ by looking up $F$ critical value in $F$-distribution table
- $F$-distribution parametric: $F(\text{numerator df}, \text{denominator df})$
- $\alpha$ is area to right of $F$ critical value (one-tailed test)
- $F$ and $t$ are distributions are related: $F(1, q) = t(q)^2$

From [Saville Wood 91], p 52, and [Devore Peck 86], p 563
ANOV A Example

• Hypothesis $H_1$:
  – Platform (Workbench, Desktop, Cave, or Wall) will affect user navigation time in a virtual environment.

• Null hypothesis $H_0$: $\mu_b = \mu_d = \mu_c = \mu_w$.
  – Platform will have no effect on user navigation time.

• Ran 32 subjects, each subject used each platform, collected 128 data points.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>$F$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between (platform)</td>
<td>1205.8876</td>
<td>3</td>
<td>401.9625</td>
<td>3.100*</td>
<td>0.031</td>
</tr>
<tr>
<td>Within (P x S)</td>
<td>12059.0950</td>
<td>93</td>
<td>129.6677</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Reporting in a paper: $F(3, 93) = 3.1, p < .05$

Data from [Swan et al. 03], calculations shown in [Howell 02], p 471
Main Effects and Interactions

• Main Effect
  – The effect of a single independent variable
  – In previous example, a main effect of platform on user navigation time: users were slower on the Workbench, relative to other platforms

• Interaction
  – Two or more variables interact
  – Often, a 2-way interaction can describe main effects

From [Howell 02], p 431
Example of an Interaction

• Main effect of drawing style:
  – $F(2,14) = 8.84, p < .01$
  – Subjects slower with wireframe style

• Main effect of intensity:
  – $F(1,7) = 13.16, p < .01$
  – Subjects faster with decreasing intensity

• Interaction between drawing style and intensity:
  – $F(2,14) = 9.38, p < .01$
  – The effect of decreasing intensity occurs only for the wireframe drawing style; for fill and wire+fill, intensity had no effect
  – This completely describes the main effects discussed above

Data from [Living et al. 03]
Reporting Statistical Results

• For parametric tests, give degrees of freedom, critical value, p value:
  – \( F(2,14) = 8.84^*, p < .01 \) (report pre-planned significance value)
  – \( t(8) = 4.11, p = .0034 \) (report exact p value)
  – \( F(8,12) = 5.826403, p = 3.4778689e10^{-3} \)
    (too many insignificant digits)

• Give primary trends and findings in graphs
  – Best guide is [Tufte 83]

• Use graphs / tables to give data, and use text to discuss what the data means
  – Avoid giving too much data in running text
References


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Slide Location:
Classical and Other Psychophysical Methods for Virtual Environments

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Advanced Controls & Displays Group
NASA Ames Research Center

Human Systems Integration Division
Outline

• Motivation: VE Latency/Asynchrony characterization
• Psychophysics: What and why?
• Classical methods of psychophysics
  – Method of Constant Stimuli
    • Detection theory
  – Method of Limits
    • Up-Down procedures
• Adaptive methods of psychophysics
• Psychometric function

Illustrations from NASA-Ames studies
Temporal & Spatial Imperfection in (Visual) VEs

**Excessive time delay and insufficient frame (update) rate**

- Poor dynamic registration, dynamic instability
- “Sloshiness,” jumpiness in response to observer motion
  Whole image lags in response to head motion

Systematic and random error in spatial measurement

- Poor static registration wrt external world
- VE image jitter

**Degraded motor and perceptual performance**

- Diminished interactivity, immersion, & sense of “presence”
- “Cybersickness”
Latency Induced Rendering Errors

6 Frames Delay
20 Hz Update Rate
~ 380 ms Latency
Latency/Asynchrony Studies
ADSP/ACD Group

• Phenomenon: Tracking & tracing performance (latency & update rate)
• First quantification of VE head and hand latency perception MoCS
• Compensation techniques: perceptually based design validation MoCS
• Latency perception mechanism: direct time vs. image “slip” MoL
• How we perceive image “slip”: displacement vs velocity AS
• Generalizability of perceptual threshold quantification AS
• Why we perceive image “slip” velocity

• Haptic-audio asynchrony thresholds AS
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Definition

• Psychophysics:
  – Area of psychology that employs specific behavioral methods to study the relation between the physical world and subjective experience (after S. Lederman)

  – Quantitative evaluation of perceptual characteristics (e.g., sensitivity) as a function of physical stimulus parameters

• Empirical, Analytical, Theoretical
The Questions

• What is it?
  – A priori; qualitative

• Is it there?
  – Absolute threshold (RL)

• How different is it [than standard]?
  – Differential threshold (DL)

• How much is there?
  – Magnitude estimation
Why Psychophysics for/in VE?

• Quantify perceptual tolerances that are relevant to Virtual Environment (VE) system use
  – Establish guidelines and specifications for the design, implementation, and effective deployment of VE systems and interfaces
• Ultimately, to use appropriately implemented and well calibrated VE systems to rapidly prototype psychophysical (and other performance) studies
• We want to measure human performance, not system artifact!
(Classical) Psychophysical Methods

- Method of Adjustment
- Method of Constant Stimuli
- Method of Limits
  - Staircases
  - Up-down Staircases
  - Adaptive Staircases
Method of Adjustment

- Observer adjusts a stimulus
  - to exceed a threshold ($RL$): absolute threshold
  - to match a standard ($DL$): difference threshold

- Example
  - Manually (literally or figuratively) adjust an apparatus setting (e.g., by turning a knob) until a temporal or spatial (or other intensity) separation is (or is no longer) {heard|felt|seen} between sequentially presented stimuli
Method of Constant Stimuli

- Intervals presented
  - \( N \) (noise)
    - absence of stimulus, reference condition, standard
  - \( S+N \) (signal plus noise)
    - stimulus, probe condition

- Depending on the stimulus type, intervals are presented
  - individually (single interval) for absolute threshold (RL)
    - yes|no response
  - pairs (two-interval)
    - simultaneously in adjacent locations, sequentially in same or adjacent location
      - which interval is bigger|smaller?
  - \( n \)-interval
Method of Constant Stimuli

- Response: Two Alternative Force Choice (2AFC)
- Q: Is signal (S) present?
- Other designs are possible

<table>
<thead>
<tr>
<th>Stimulus</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>S+N</td>
<td>Hit</td>
</tr>
<tr>
<td>N</td>
<td>False Alarm</td>
</tr>
</tbody>
</table>

Type I error: N
Type II error: Miss
Detection Theory: Internal Response

- Criterion is individual observer’s preference or bias; depends on cost/pay-off
Discriminability: $d'$ (d-prime)

- Assumptions
  - $N, S+N$ are Gaussian
  - $N, S+N$ have equal variance
Discriminability: $d'$ (d-prime)

- Assumptions
  - $N$, $S+N$ are Gaussian
  - $N$, $S+N$ have equal variance
- For Z (normal) distribution ($\sigma_n = 1$):
  \[ d' = Z_H - Z_{FA} \]
Discriminability: $d'$ (d-prime)

- Assumptions
  - $N, S+N$ are Gaussian
  - $N, S+N$ have equal variance
- For Z (normal) distribution ($\sigma_n = 1$):
  \[ d' = Z_H - Z_{FA} \]
- Possibility of criterion (bias) shift with constant $d'$
ROC: Receiver Operating Characteristic (AKA Relative Operating Characteristic)

Increased $d'$ → improved discriminability
ROC: Receiver Operating Characteristic (AKA Relative Operating Characteristic)

Increased $d' \rightarrow$

improved discriminability

(threshold $d' > 1$)
ROC: Receiver Operating Characteristic (AKA Relative Operating Characteristic)

Increased $d' \rightarrow$ improved discriminability (threshold $d' > 1$)

ROC: $p(H)$ vs $p(FA)$ as function of $d'$
Example: Latency Discrimination
Constant Stimuli Experiment [1]

Experiment Factors (3 X 7 levels)

$T = \{33, 100, 200\}$ ms
$\Delta t = \{16.7, 33.3, \ldots, 116.7\}$ ms

Randomized Stimulus Block

Q (2AFC): Same or different?
Example: Latency Discrimination
Hit/FA rates and d-prime

(One Observer)
Example: Latency Discrimination
Average Hit/FA rate and d-prime

(8 Observers)
Hit/FA Rates vs. Stimulus

• Ideally we want low and uniform $p(FA)$
  – Reliability in performing the judgment task
  – Constant criterion; no drift in bias
  – $d'$ depends on hit rate, $p(H)$
• $p(H)$ as a function of stimulus intensity
  – Psychometric function
• Thresholds and bias
  – More later w/ Methods of Limits
(Truncated) Method of Limits

- Staircases (non-reversing) algorithm
  - Define stimulus range
  - Start high: descend until “Not Different”
  - Start low: ascend until “Different”
- 50% threshold
Example: Latency Discrimination
Staircase Experiment [2]

$T = \{33, 100, 200\} \text{ ms}$

Experiment Factor (3 levels)

Staircases start either

LOW
$\Delta t = 0 \text{ ms (randomly 1 to 3 times)}$
and increase until “different”
or
HIGH
$\Delta t = \{116.7, 133.3, 150.0\} \text{ ms (randomly selected)}$
and decrease until “not different”

Q (2AFC): Same or different?
Example: Latency Discrimination
Staircase Experiment

Threshold (50%) = 77.5 ms
20 runs; 131 pts

Added Latency (ms)

Detection Rate

Run #

Obs: JM
Ref: 33 ms
Pace: const

Obs: JM
Ref: 33 ms
Pace: const

Added Latency (ms)
Example: Latency Discrimination
Staircase Experiment

• Staircases (non-reversing)
  – Each staircase yields a termination level
  – From which can reconstitute raw staircase data
  – Construct $p(H)$ as a function of stimulus intensity
Method of Limits

• Simple truncated method of limits up–down method or:

\[ x_{n+1} = x_n - \delta (2 z_n - 1) \]

\{\text{miss|hit}\}: \ z_n = \{0|1\}

• Fixed step size \( \delta \)
• Avoids stimulus presentation far above and below threshold
Method of Limits
Up-Down Staircases

• Transformed Up-Down staircases w/ or w/o adaptation

• 1 Up-$N$ Down staircase theoretical convergence levels (“equilibrium”): $0.5 = 1 - p(H)^N$
  – 1U-1D: 50.0%
  – 1U-2D: 70.7% (Transformed)
  – 1U-3D: 79.4% (Transformed)

• Analytic relation of $d'$ to equilibrium for 1U-ND staircase and $M$-Alternative Force Choice
  – Use $d'$ to help choose staircase method
Example: Asynchrony Discrimination
1U-2D Adaptive Staircase Experiment [3]

\[ T = \{7.2\} \text{ms} \]

Staircases start either LOW \( \Delta t = 0 \text{ ms} \) or HIGH \( \Delta t = 256 \text{ ms} \)

\[ \min(\Delta t) = 4 \text{ ms} \]

Q (2AFC): **Which** (1 or 2) was “Reference”?
Example: Asynchrony Discrimination
1U-2D Adaptive Staircase Experiment
(70.7\% Threshold)
Example: Asynchrony Discrimination

Adaptive Staircase Experiment

• Each staircase *ideally* converges to an equilibrium level, corresponding to a theoretical threshold
  – Drift (criterion shift) with extended duration
• Construct \( p(H) \) as a function of stimulus intensity
• Adaptive staircases
  – Focus most quickly on region of interest
  – More on region of interest in section on Psychometric Function
• Interleaved staircases
  – Prevent observer tracking/prediction
Psychometric Function

• Construct a model (i.e., psychometric function) describing relation between input stimulus intensity and observer’s detection/discrimination rate.

• Looking for best fit of given function to experimentally measured data through optimization of model parameter space.

• In the following illustrations, Gaussian distributions (i.e., two parameter model: $\mu$ and $\sigma$) are fitted to minimization of weighted least-square ($\chi^2$) error.
Psychometric Functions

• Some typical monotonically increasing functions

Gaussian distribution: \( N(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{x} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \)

Logistic distribution: \( L(x; \alpha, \beta, \gamma) = \frac{1}{1 + \exp\left(\frac{\alpha - x}{\beta}\right)} \)  
\( (\beta = \sigma/1.7; \alpha = \mu) \)  
(Gaussian approx.)

Weibull distribution: \( W^{(1)}(x; \alpha, \beta, \gamma) = 1 - \exp\left(\frac{(x-\gamma)^\beta}{\alpha}\right) \)
Psychometric Function

- Features of the ogive

Point of Subjective Equality (PSE) and bias with respect to reference stimulus

Just Noticeable Difference (JND) for psychometric function symmetric about PSE
Psychometric Function

• Features of the ogive
  – Point of Subjective Equality/Equivalence (PSE)
    • Bias in observer’s response
    • Criterion dependent
    • Question posed as a source of bias
  – Just Noticeable Difference (JND)
    • Generally defined by $\frac{1}{2}$ of stimulus difference between 1st and 3rd detection quartiles
    • For symmetric functions, the amount of additional stimulus difference to increase detection by 25% from PSE
    • JND is related to variance and is therefore a statistical measure of detectability
Fitting a Psychometric Function

• Practical considerations (for standard normal model)
  – Transform data to standard normal (Z) coordinates and apply linear regression
  – *Probability paper (cf. semi-log paper)*
  – Functional fit minimizing weighted error of fit to data
    • Weighted by binomial standard error for fitted model (Probit with $\chi^2$ error/model)
  – “Finger error”:
    • Rates of guessing ($p_g$); rates of lapsing ($p_l$)
    • Alleviates problem of $P = 0$ or 1, i.e., $Z \rightarrow -\infty$ or $\infty$
Psychometric Functions
Constant Stimuli Study [1]

Fitted to Cumulative Gaussian Distribution
Psychometric Functions
Staircase Studies [2],[3]

50% threshold = 77.5 ms

75% level = 41.1 ms
70.7% threshold = 36.7 ms
Just-Noticeable Differences (JND): 12 Observers


Group 1

Group 2

Base: \( F(2,20) = 4.044 \quad p < .022 \)
Group X Epoch X Base: \( F(2,20) = 4.866 \quad p < .019 \)
Summary Comments on Methods

• Method choice should depend on objectives

• Use Method of Constant Stimuli first, when have insufficient knowledge of detection capacity
  – Measure $d$-prime and FA rates
  – Time-consuming (inefficient)

• Method of Limits w/ U-D Adapting Staircases
  – Can select U-D ratio to concentrate data in region of interest
    • Efficient (fewer observations than Constant Stimuli)
  – Does not measure FA rate
    • Has a prescribed $d'$ for given $M$-alternative forced choice

Caveat: Pure perception experiments may be far removed from ecological experience—i.e., detached from realistic action and task performance
Summary Comments on Methods

• Method choice should depend on objectives

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References

http://human-factors.arc.nasa.gov/ihh/spatial/papers.html


Further Reading


Human Performance and Preference Studies
Stephen R. Ellis
Ames Research Center
Moffett Field, CA USA

Outline
1. Purpose and Human Performance assessment
2. Example: Interface preference data
3. Measurement
   3.1 Stevens's classification of measurement
   3.2. Critique of Stevens's classification
4. Three Illustrative Cases Studies
   4.1. Nominal data: Maneuver distributions
   4.2. Ordinal data: correlation, Friedman ANOVA
   4.3. Interval data: ANOVA
5. Some Heuristics for Behavioral Data Analysis
The purpose of a human performance assessment within a virtual environment is to determine whether the virtual environmental users are able to realize the goals and expectations they bring upon entering it without unacceptable costs and risks.

Seek information that is
1. True
2. Reliable
3. Valid
4. Knowably generalizeable
5. Task appropriate
Exhortation #1

Think and Argue Causally
### Historical Display of Orbiter O-ring Damage

#### History of O-Ring Damage in Field Joints (Cont)

<table>
<thead>
<tr>
<th>O-Ring Temp (°F)</th>
<th>SRM No.</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>66°</td>
<td>1</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>70°</td>
<td>1</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>66°</td>
<td>2</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>69°</td>
<td>2</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>80°</td>
<td>3</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>68°</td>
<td>3</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>67°</td>
<td>4</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>72°</td>
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<td>B</td>
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<td>73°</td>
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<td>A</td>
<td>B</td>
</tr>
<tr>
<td>70°</td>
<td>5</td>
<td>A</td>
<td>B</td>
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<td>67°</td>
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<td>A</td>
<td>B</td>
</tr>
<tr>
<td>72°</td>
<td>6</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>63°</td>
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<td>A</td>
<td>B</td>
</tr>
<tr>
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<td>7</td>
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<td>B</td>
</tr>
<tr>
<td>67°</td>
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<td>A</td>
<td>B</td>
</tr>
<tr>
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<td>B</td>
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<td>63°</td>
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<td>B</td>
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<td>B</td>
</tr>
<tr>
<td>67°</td>
<td>12</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

**Note:**
- No Erosion
Explanatory Display of Orbiter O-ring Damage

Expected temperature range for Challenger launch

Temperature (°F) of field joints at time of launch

O-ring damage index

SRM 15

SRM 22

( after Tufte, 1997)
Exhortation #2
Consider Alternative Investigative Approaches
How is the evaluation done?

**Theoretical**
- Mathematical
- Logical
- Computational

**Empirical**
- Observational
- Longitudinal
- Cross-sectional
- Experimental
  - Subject is own control.
- Independent groups
- Repeated measures
- Meta-analytic
Exhortation #3

Behavioral

“Measurements” Sometimes Yield Surprising Paradoxes
Lickert Scale Opinion Assessment I: questionnaire

User interface options: option layout in menus

<table>
<thead>
<tr>
<th>Fixed</th>
<th>Static Match</th>
<th>Dynamic Matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed menu</td>
<td>Usage frequency</td>
<td>Usage frequency</td>
</tr>
<tr>
<td>sequence</td>
<td>adjusted menu sequence</td>
<td>menu sequence:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>dynamically adjusted</td>
</tr>
</tbody>
</table>

- written instructions, training
- laptop based data collection
- repeated measures, randomized presentation, order balancing, +

Awful, Totally unusable

0 1 2 3 4

1.82

Perfect, No improvements needed
Lickert Scale Opinion Assessment II: preference scores

Preference data (Interval:mean)

<table>
<thead>
<tr>
<th></th>
<th>Fixed</th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.61</td>
<td>2.76</td>
<td>2.04</td>
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<tr>
<td>SE</td>
<td>0.2214</td>
<td>0.2359</td>
<td>0.2303</td>
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</tbody>
</table>
Lickert Scale Opinion Assessment III: ANOVA

Repeated measures ANOVA

<table>
<thead>
<tr>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square (variance)</th>
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<tbody>
<tr>
<td>SSqr (total)</td>
<td>43.3185</td>
<td></td>
</tr>
<tr>
<td>SSqr between</td>
<td>10.5103</td>
<td>5.255</td>
</tr>
<tr>
<td>SSqr within</td>
<td>32.8081</td>
<td>1.172</td>
</tr>
</tbody>
</table>

\[ F = 4.485 \]
\[ F(\text{crit}, .05) = 3.340 \]
\[ F(\text{crit}, .025) = 4.221 \]
Lickert Scale Opinion Assessment IV: rank transforms

1 ~ least preferred    3 ~ most preferred

Preference data (Rank:median)

Friedman ANOVA

\[ \chi_r^2(2) = 0 \]
Not significant

Median
sterr SIQR
Fixed 2 0.2031
Static 2 0.2031
Dynamic 2 0.2031
K. Arrow Voting Paradox with Heterogeneous Ordinal Preferences

<table>
<thead>
<tr>
<th>Subjects</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>a &lt; b</th>
<th>b &lt; c</th>
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<td>1</td>
<td>2</td>
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<td>2</td>
<td>3</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Vote:

No: 5
Yes: 10

1 ~ Yes " " " ~ No ?

5

10
Exhortation #4

A number is not always a number!
Measurement

The systematic assignment of scale values, usually numbers, to observations or objects with the purpose of representing and modeling the measured entities.

Some desirable properties of measurements

1. Public
2. Unique
3. Knowably precise
4. Reliable & stable
5. Robust
6. Valid
<table>
<thead>
<tr>
<th>Scale</th>
<th>Property</th>
<th>Symmetry/Operation (inverse)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>identity</td>
<td>Replacement (Replacement)</td>
</tr>
<tr>
<td>Ordinal</td>
<td>rank</td>
<td>Sorting (Unsorting)</td>
</tr>
<tr>
<td>Interval</td>
<td>unit</td>
<td>Aggregation (Disaggregation)</td>
</tr>
<tr>
<td>Ratio</td>
<td>zero</td>
<td>Multiplication (Division)</td>
</tr>
</tbody>
</table>

Variety of Measurement Scales due to Stevens

Nominal, Ordinal, Interval, Ratio

Quality, Equivalence class, Quantity

(Stevens, 1946)
The Meaning of an Equivalence Class

(Ellis, 1996)
Equivalence Classes and Explanation

Presence measure

Consequent Phenomenon
# Measurement Scales and Appropriate Statistics

<table>
<thead>
<tr>
<th>Scale</th>
<th>Property</th>
<th>Allowable Transformations</th>
<th>Associated Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>identity</td>
<td>Renaming</td>
<td><strong>Mode</strong> = max[frequenc($x_i$)]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Index of variety</strong> = - $\Sigma$ prob($x_i$)log(prob($x_i$)) (bits)</td>
</tr>
<tr>
<td>Ordinal</td>
<td>rank</td>
<td>Monotonic transformation</td>
<td><strong>Contingency correlation</strong> = $\sqrt{\frac{X^2}{N(k-1)}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Preserving order</td>
<td>$X^2 = \sum \frac{o_i^2}{e_i} - N$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\max(X^2) = N(k-1)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Median</strong> = percentile$_{50}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Interquartile range</strong> = percentile$<em>{75}$ - percentile$</em>{25}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Rank order Spearman correlation</strong> = 1 - $\frac{\sum d_i^2}{N^2 - N}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Friedman ANOVA</strong></td>
</tr>
<tr>
<td>Interval</td>
<td>unit</td>
<td>Linear transformation preserving differences to a scale factor</td>
<td><strong>Mean</strong> = $\frac{\sum X_i}{N}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Standard deviation</strong> = $\frac{\sum (X_i - \overline{X})^2}{N}$, $\sum (X_i - \overline{X}) (Y_i - \overline{Y})$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Product-moment correlation</strong> = $\frac{\sqrt{\sum (X_i - \overline{X})^2 \sum (Y_i - \overline{Y})^2}}$</td>
</tr>
<tr>
<td>Ratio</td>
<td>zero</td>
<td>Nonlinear Transformations preserving ratios to a scale factor.</td>
<td><strong>Mean</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>Standard deviation</strong></td>
</tr>
<tr>
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<td></td>
<td><strong>Product-moment correlation</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>ANOVA</strong></td>
</tr>
</tbody>
</table>
Pros & Cons of Stevens’s Measurement Classification

Pros

1. Discourage use of measurement properties implicit in numerical measurement but not necessarily supported by the measurement technique. *Tells what kinds of difference make a difference!*  
   
2. Reinforces due consideration of the assumptions underlying conventional statistical processing, i.e. sampling, distribution, variance  
   
3. Potential for algorithmic or heuristic control of data analysis.  
   
4. Can be an aid for selecting appropriate statistics for analysis.  

Cons

1. *A. priori* data typing may preclude serendipitous discovery.  
   
2. Stevens’s scale categorization are absolute resulting in demoting to lower scales resulting in loss of information  
   
3. Statistics should be selected based on what kinds of questions we ask of the data not properties of the data themselves.  
   
4. Potential for algorithmic or heuristic control of data analysis.
Nominal Data: Cockpit Traffic Display Based Avoidance Maneuvers

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Counted</th>
<th>Ho</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horiz.</td>
<td>Vertical</td>
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<td></td>
<td>76</td>
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<td></td>
<td>32</td>
<td>32</td>
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<td></td>
<td>108</td>
<td>34</td>
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</table>

<table>
<thead>
<tr>
<th>Expected freq. ( f_e ), all ( f_e ) &gt;&gt; 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counted</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

\[
X^2 = \sum \frac{(o_i - e_i)^2}{e_i}
\]

\[
X^2 = \frac{(8.963 - 13.235)^2}{13.235} + \frac{(8.963 - 13.235)^2}{13.235} + \frac{(1.96 - 1.96)^2}{1.96} = 48.317
\]

\[
X^2(2) > 48.317 > 13.82, \text{ critical } X^2(2)@p < 0.001
\]
Spearman Rank Order Correlation: $r_s$

Measure of correlation for data that are only meaningful in terms of order, derived from the standard product moment correlation, $r$, i.e. $r_s = r$ of the data reduced to ranks for $N$ pairs of correlated variables $x$, $y$, with mean ranks $X$ and $Y$ and rank differences $d_i$.

$$r_s = r = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \text{var}(y)}} = \frac{\sum_i (x_i - X)(y_i - Y)}{\sqrt{\sum_i (x_i - X)^2 \sum_i (y_i - Y)^2}} \leq 1$$

$$r_s = 1 - \frac{6 \sum d_i^2}{N(N^2 - 1)}$$

$$t = \frac{r_s \sqrt{N - 2}}{\sqrt{1 - r_s^2}}, N \gg 10$$
### Spearman Rank Order Correlation: $r_s$

<table>
<thead>
<tr>
<th>Subjects</th>
<th>Cooper-Harper</th>
<th>Stability</th>
<th>Mod C/H rank</th>
<th>SS rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0</td>
<td>3.0</td>
<td>4.5</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>4.0</td>
<td>1.5</td>
<td>4.5</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>4.2</td>
<td>Ties are assigned the average of ranks otherwise assigned.</td>
<td>8.5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3.5</td>
<td>1</td>
<td>4.5</td>
<td>7</td>
</tr>
<tr>
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<td>4.0</td>
<td>2.0</td>
<td>4.5</td>
<td>7</td>
</tr>
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<td>3.0</td>
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<td>3.0</td>
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<td>7</td>
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<tr>
<td>12</td>
<td>4.0</td>
<td>4.5</td>
<td>4.5</td>
<td>11</td>
</tr>
</tbody>
</table>

$r_s = 0.425\text{ns}$ $r = 0.625^*$, $df = 10$, $^*\text{crit}(0.05) = 0.576$
Subdivisions of a random selection of sample statistics should provide estimates of the same population parameter if the classification into subgroups has no effect on subgroup statistics.
Example of One way Independent Groups ANOVA

Strategy: estimate a population statistic (a variance) two different ways so that if $H_o$ is true the ratio of these estimates will be 1. Significant deviations from 1, refute $H_o$ given assumption of random sampling, normal distribution, homogeneity of variance.

Notation

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>...</th>
<th>Group k</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{1,1}$</td>
<td>$x_{1,2}$</td>
<td></td>
<td>$x_{1,k}$</td>
</tr>
<tr>
<td>$x_{2,1}$</td>
<td>$x_{2,2}$</td>
<td></td>
<td>$x_{2,k}$</td>
</tr>
<tr>
<td>$x_{3,1}$</td>
<td>$x_{3,2}$</td>
<td></td>
<td>$x_{3,k}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>$x_{n1,1}$</td>
<td>$x_{n2,2}$</td>
<td></td>
<td>$x_{nk,k}$</td>
</tr>
</tbody>
</table>

Group means

- $X_1$
- $X_2$
- $X_k$

Grand Mean $X$

- $\sum_{j}n_j = N$
- $df = N - 1$

$df = n_1 - 1$  $df = n_2 - 1$  etc
One-way ANOVA: Partitioning Sums of Squares (SS) and Definition of Mean Squares (MS) variance estimates (R. Fisher)

For the jth group

\[(x_{ij} - X) = (x_{ij} - X_j) + (X_j - X)\]

identity

\[\sum_i (x_{ij} - X)^2 = \sum_i (x_{ij} - X_j)^2 + \sum_i (X_j - X)^2 + 2 (X_j - X) \sum_i (x_{ij} - X_j) \]

sqr, sum

\[\sum_i (x_{ij} - X)^2 = \sum_i (x_{ij} - X_j)^2 + n_j (X_j - X)^2\]

summation of constant, dev sum to 0

\[\sum_j \sum_i (x_{ij} - X) = \sum_j \sum_i (x_{ij} - X_j) + \sum_j n_j (X_j - X)^2\]

sum over k groups

Total SS = SS\text{within groups} + SS\text{between groups}

definition

\[df\text{within} = (n_1 - 1) + (n_2 - 1) + \ldots + (n_k - 1) = \sum_j n_j - k = N - k\]

df\text{between} = k - 1

\[MS\text{within} = \frac{SS\text{within}}{df\text{within}}, \quad MS\text{between} = \frac{SS\text{between}}{df\text{between}}\]

definition

F statistic with k-1, N-k degrees of freedom = \[\frac{MS\text{between}}{MS\text{within}}\]
Independent Groups ANOVA

F = 5.464
F(crit, 0.05) = 3.592
Example of Friedman Nonparametric ANOVA

Lickert Scale Opinion Assessment IV: rank transforms

1 ~ least preferred    3 ~ most preferred

<table>
<thead>
<tr>
<th>Subjects</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>15</td>
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<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ J = 3 \]

\[ T_j = 30 \quad 30 \quad 30 \]

\[ \chi_r^2 = \frac{12}{KJ(J+1)} \sum T_i^2 - 3K(J+1) \]

\[ 0 \leq \chi_r^2 \leq K(J-1), \quad df = J-1, \]

If \( J > 3 \) and \( K > 9 \), use \( \chi^2 \)

Otherwise use tables for \( \chi_r^2 \)

0.033, ns
Nonparametric Statistics: Pros & Cons

Pros

1. No assumed population normality or homogeneity of variance.

2. Even data that may be higher order than ordinal may be evaluated with relaxed statistical assumptions.

3. Some nonparametric tests may be used with very small sample sizes (~5) and provide exact probabilities (e.g. binomial test)

4. Nonparametric tests may be applied to nominal data for which there are no alternatives.

5. Simpler to calculate, may aid intuition about size of effects.

Cons

1. Nonp samp  
   Power-efficient of test_B = (100) N_A/N_B

2. Nonp (e.g. 
   N_B = Observations for the same power for test_B)

3. Converting data to ranks throws away scientifically interesting ordinal or ratio information.
Some Heuristics for Behavioral Experimentation

In General
- Statistics are ideally descriptive and reinforce results evident by plots and model fits, the goals of an experiment are *data and models, not statistics*.
- Review handbooks/design and user performance reference material before starting.

About Methods
- *Placebo* and *Hawthorne* effects are real: consider a variety of control groups.
- Use balanced independent groups for major independent variables when possible, distribute group assignment over experimental run.
- Evaluate behavior related to closed-loop performance.
- Check statistical assumptions when possible, i.e. normality, at least unimodality, symmetry and equality of variance.

About Results and Conclusions
- Results should not be dependent upon a specific measurement scale
- Results should be robust to exclusion of outliers.
- Statistical conclusions should not depend upon a specific analytic approach.
References


End